MATH 111: HOMEWORK 01 SOLUTIONS

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A.2

3. *The set of numbers between but not including* 2 *and* 7 *can be written as follows:*

Solution. (a) $\{x \in \mathbb{R} \mid 2 < x < 7\}$ in set builder notation.
(b) $(2, 7)$ in interval notation.

4. *Explain the differences between the following two sets:* $A = [-2, 5]$ *and* $B = (-2, 5)$.

Solution. The set A contains the endpoints, 2 and -5, but the set B does not.

27 & 29: Find the indicated set if $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 4, 6, 8\}$, and $C = \{7, 8, 9, 10\}$.

27. *(a)* $A \cup B$ *(b)* $A \cap B$

Solution. (a) The union of A and B is the set containing the elements of both sets,

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}.$$

(b) The intersection of A and B is the set containing the elements common to both sets,

$$A \cap B = \{2, 4, 6\}.$$

29. *(a)* $A \cup C$ *(b)* $A \cap C$

Solution. (a) The union of A and C is the set containing the elements of both sets,

$$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

(b) The intersection of A and B is the set containing the elements common to both sets,

$$A \cap C = \{7\}.$$

49. *Express the inequality in interval notation, and then graph the corresponding interval.*

$$-5 \le x < -2.$$

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Solution. The inequality $-5 \leq x < -2$ represents the half-open interval that includes the endpoint -5, but not the endpoint -2 and so can be written $[-5, 2)$.

63. *Find the distance between the given numbers.*

(a) 2 *and* 17
(b) -3 *and* 21
(c) $\frac{11}{8}$ *and* $-\frac{3}{8}$.

Solution. (a) The distance between 2 and 17 is given by

$$\mathrm{d}(2, 17) = |2 - 17| = |-15| = 15.$$

(b) The distance between -3 and 21 is giveb by

$$\mathrm{d}(-3, 21) = |-3 - 21| = |-24| = 24.$$

(c) The distance between $11/8$ and $-3/8$ is given by

$$\mathrm{d}\left(\frac{11}{8}, -\frac{3}{8}\right) = \left|\frac{11}{8} - \left(-\frac{3}{8}\right)\right| = \left|\frac{11+3}{8}\right| = 14/8 = 7/4.$$

A.3

1. *Using exponential notation, we can write the product* $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ *as* ________.

Solution. This is the product of 5 with itself 6 times, we we can write it as 5^6.

3. *When we multiply two powers with the same base, we* ________ *the exponents. So* $3^4 \cdot 3^5 =$ ________.

Solution. When we multiply two powers with the same base, we add the exponents, so

$$3^4 \cdot 3^5 = 3^{4+5} = 3^9.$$

6. *Express the following without using exponents.*
(a) $2^{-1} =$ ________.
(b) $2^{-3} =$ ________.
(c) $\left(\frac{1}{2}\right)^{-1} =$ ________.

Solution. When a number is raised to a negative exponent, this is the same as raising its reciprocal to the absolute value of the exponent. Hence the solutions are

(a) $2^{-1} = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$,

(b) $2^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1}{2^3} = \frac{1}{8}$,

(c) $\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$.

29 & 33: Use the rules of exponents to write each expression in as simple a form as possible.

29. $\dfrac{7^5 \cdot 7^{-3}}{7^2}$.

Solution. Simplifying the numerator, we add the exponents in the product to obtain $7^5 \cdot 7^{-3} = 7^{5+(-3)} = 7^2$. Therefore the ratio reduces to

$$\frac{7^5 \cdot 7^{-3}}{7^2} = \frac{7^2}{7^2} = 1.$$

33. $\left(\frac{1}{2}\right)^4 \left(\frac{5}{2}\right)^{-2}$.

Solution. Expanding each ratio in turn we have

$$\left(\frac{1}{2}\right)^4 = \frac{1}{2^4} = \frac{1}{16}$$

and

$$\left(\frac{5}{2}\right)^{-2} = \left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2} = \frac{4}{25}.$$

Combining these calculations we obtain

$$\left(\frac{1}{2}\right)^4 \left(\frac{5}{2}\right)^{-2} = \frac{1}{16}\frac{4}{25} = \frac{1}{4 \cdot 25} = \frac{1}{100}.$$

52. *Simplify the expression, and eliminate any negative exponents:*

$$\left(\frac{-2x^2}{y^3}\right)^3.$$

Solution. Using the rules of exponentiation we have

$$\left(\frac{-2x^2}{y^3}\right)^3 = \frac{(-2)^3(x^2)^3}{(y^3)^3} = \frac{-8x^{2\cdot 3}}{y^{3\cdot 3}} = \frac{-8x^6}{y^9}$$

A.4

1. *Using exponential notation, we can write* $\sqrt[3]{5}$ *as* ______.

Solution. The third root of 5, $\sqrt[3]{5}$, can be written using rational exponents as $5^{1/3}$.

4. *Explain what* $4^{3/2}$ *means, and then calculate* $4^{3/2}$ *in two different ways:*

$$4^{3/2} = \left(4^{1/2}\right)^{\square} = \_\_\_\_\_\_ \qquad 4^{3/2} = \left(4^{3}\right)^{\square} = \_\_\_\_\_\_$$

Solution. The symbol $4^{3/2}$ means the cube of the square root of 4, written $(\sqrt{4})^3$, or, equivalently, the square root of the cube of 4, written $\sqrt{4^3}$. Using exponential notation, these can be written as $\left(4^{1/2}\right)^3$ and $(4^3)^{1/2}$, respectively.

17. *Evaluate the expressions*

(a) $\sqrt{\frac{4}{9}}$,

(b) $\sqrt[4]{256}$,

(c) $\sqrt[6]{\frac{1}{64}}$.

Solution. (a) Using the rules of exponents, we have

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}.$$

(b) First we observe that $256 = 2^8$. Then using the rules of exponents, it follows that

$$\sqrt[4]{256} = \sqrt[4]{2^8} = 2^{8/4} = 2^2 = 4.$$

(c) Here we note that $64 = 2^6$ so we obtain

$$\sqrt[6]{\frac{1}{64}} = \sqrt[6]{2^6} = \left(2^6\right)^{1/6} = 2^{6/6} = 2^1 = 2.$$

23, 25, & 33: simplify the expression and eliminate any negative exponent(s). Assume that all letters denote positive numbers.

23. $x^{3/4}x^{5/4}$.

Solution. Using the rules of exponents, we have

$$x^{3/4} \cdot x^{5/4} = x^{3/4+5/4} = x^{8/4} = x^2.$$

25. $(4b)^{1/2}$.

Solution. Again using the rules of exponents, we obtain

$$(4b)^{1/2} = \sqrt{4b} = \sqrt{4}\sqrt{b} = 2\sqrt{b}.$$

33. $\left(\dfrac{2q^{3/4}}{r^{3/2}}\right)^{-4}.$

Solution. Using the rules of exponents

$$\left(\frac{2q^{3/4}}{r^{3/2}}\right)^{-4} = \left(\frac{r^{3/2}}{2q^{3/4}}\right)^{4} = \frac{(r^{3/2})^4}{(2q^{3/4})^4} = \frac{r^{(3/2)\cdot 4}}{2^4 q^{(3/4)\cdot 4}} = \frac{r^6}{16q^3}.$$

41. *Simplify the expression and express the answer using rational exponents. Assume that all letters denote positive numbers.*

$$\left(5\sqrt[3]{x}\right)\left(2\sqrt[4]{x}\right).$$

Solution. Using the rules for exponents,

$$\left(5\sqrt[3]{x}\right)\left(2\sqrt[4]{x}\right) = 10x^{1/3}x^{1/4} = 10x^{1/3+1/4} = 10x^{4/12+3/12} = 10x^{7/12}.$$

B.1

1. *To add expressions, we add* _______ *terms. So*
$(3a + 2b + 4) + (a - b + 1) =$ _______.

Solution. To add expressions, we add like terms, so

$$(3a + 2b + 4) + (a - b + 1) = 4a + b + 5.$$

2. *To subtract expressions, we substract* _______ *terms. So*
$(2xy + 9b + c + 10) - (xy + b + 6c + 8) =$ _______.

Solution. To subtract expressions, we subtract like terms, so

$$(2xy + 9b + c + 10) - (xy + b + 6c + 8) = xy + 8b - 5c + 2.$$

4. *Explain how we multiply two binomials, then perform the following multiplication:*
$(x + 2)(x + 3) =$ _______ .

Solution. When mutliplying binomials, we use FOIL to expand the expression. Hence

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + (3 + 2)x + 6 = x^2 + 5x + 6.$$

5. *The Special Product Formula for the "square of a sum" is*
$(A+B)^2 =$ ______. *So* $(2x+3)^2 =$ ______ .

Solution. The Special Product Formula for the "square of a sum" is $(A+B)^2 = A^2+2AB+B^2$, so

$$(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9.$$

6. *The Special Product Formula for the "sum and difference of the same terms" is*
$(A+B)(A-B) =$ ______. *So* $(5+x)(5-x) =$ ______.

Solution. The Special Product Formula for the "sum and difference of the same terms" is $(A+B)(A-B) = A^2 - B^2$, so

$$(5+x)(5-x) = 5^2 - x^2 = 25 - x^2.$$

11. *Consider the expression* $2ax - 10a + 1$.

(a) What are the terms of the expression?

(b) Find the value of the expression if a *is* -1, b *is* 4, x *is* -2, *and* y *is* 3.

Solution. (a) The terms of the expression are $2ax$, $10a$, and 1.

(b) The value of the expression is

$$2ax - 10a + 1 = 2(-1)(-2) - 10(-1) + 1 = 4 + 10 + 1 = 15.$$

15 & 17: Find the sum or difference.

15. $(12x-7) - (5x-12)$.

Solution. The difference is

$$(12x-7) - (5x-12) = 12x - 5x + 12 - 7 = (12-5)x + 5 = 7x + 5.$$

17. $(4x^2+2x) + (3x^2-5x+6)$.

Solution. The sum is

$$(4x^2+2x)+(3x^2-5x+6) = 4x^2+3x^2+2x-5x+6 = (4+3)x^2+(2-5)x+6 = 7x^2-3x+6.$$

24. *Multiply the two expressions using the Distributive Property:* $(a+b)(x-y)$.

Solution. Using the Distributive Property on the left, we have

$$(a+b)(x-y) = (a+b)x - (a+b)y.$$

The right hand side of this equation can be expanded again by using the distributive property on the right to obtain

$$(a+b)x - (a+b)y = ax + bx - ay + by.$$

Similarly, one can start with distribution on the right and then apply it again on the right to obtain the same answer,

$$(a+b)(x-y) = a(x-y) + b(x-y) = ax - ay + bx - by.$$

29. *Multiply the algebraic expressions using the FOIL method and simplify:* $(r-3)(r+5)$.

Solution. Using FOIL, we have

$$(r-3)(r+5) = r^2 + 5r - 3r - 15 = r^2 + (5-3)r - 15 = r^2 + 2r - 15.$$

35, 36, & 45: Multiply the algebraic expressions using a Special Product Formula and simplify.

35. $(x+3)^2$.

Solution. Using the square of a sum formula, we have

$$(x+3)^2 = x^2 + 6x + 9.$$

36. $(x-2)^2$.

Solution. Using the square of a difference formula, we have

$$(x-2)^2 = x^2 - 2x + 4.$$

45. $(x+5)(x-5)$.

Solution. Using the difference of squares formula, we have

$$(x+5)(x-5) = x^2 - 25.$$

61. *Find the product of the polynomials:* $(x+2)(x^2+2x+3)$.

Solution. By distribution on the left, we have

$$(x+2)(x^2+2x+3) = (x+2)x^2 + (x+2)2x + (x+2)3.$$

Then applying distribution on the right,

$$\begin{aligned}(x+2)x^2 + (x+2)2x + (x+2)3 &= x^3 + 2x^2 + 2x^2 + 4x + 3x + 6\\ &= x^3 + (2+2)x^2 + (4+3)x + 6\\ &= x^3 + 4x^2 + 7x + 6.\end{aligned}$$

B.2

1. *Consider the polynomial* $2x^5 + 6x^4 + 4x^3$.

(a) How many terms does this polynomial have?

(b) List the terms.

(c) What factor is common to each term?

(d) Factor the polynomial: $2x^5 + 6x^4 + 4x^3 =$_______.

Solution. (a) This polynomial has three terms.

(b) The terms are $2x^5$, $6x^4$, and $4x^3$.

(c) The factor common to each is $2x^3$.

(d) The polynomial factors as

$$\begin{aligned}2x^5 + 6x^4 + 4x^3 &= 2x^3(x^2+3x+2)\\ &= 2x^3(x+2)(x+1).\end{aligned}$$

2. *To factor the trinomial* $x^2+7x+10$, *we look for two integers whose product is* _______ *and whose sum is* _______. *These integers are* _______ *and* _______, *so the trinomial factors as* _______.

Solution. To factor the trinomial $x^2+7x+10$, we look for two integers whose product is 10 and whose sum is 7. These integers are 2 and 5, so the trinomial factors as $(x+2)(x+5)$.

5. *Factor* $5a - 20$.

Solution. The only factor common to $5a$ and 20 is 5, so we have the factorization

$$5a - 20 = 5(a-4).$$

7. *Factor* $30x^3 + 15x^4$.

Solution. Since $30 = 2 \cdot 15$, and both terms contain an x^3, we have the factorization

$$30x^3 + 15x^4 = 15x^3(2 + x).$$

9. *Factor* $-2x^3 + 16x$.

Solution. Factoring a $-2x$ out of both terms, we have

$$-2x^3 + 16x = -2x(x^2 - 8).$$

21. *Factor the trinomial* $y^2 - 8y + 15$.

Solution. Here we need two integers that multiply to 15 and add to -8. These integers are -5 and -3, so we have

$$y^2 - 8y + 15 = (y - 5)(y - 3).$$

25. *Factor the trinomial* $5x^2 - 7x - 6$.

Solution. Here, since we the leading coefficient is not 1, we need to consider a factorization of the form $(ax+r)(bx+s) = abx + (as+br)x + rs$. We must have $ab = 5$, so our factorization must look like $(5x+r)(x+s)$. Hence we need only decide on integers r, s such that $5s+r = -7$ and $rs = -6$. Choosing $s = -2$ and $r = 3$, we see that $5(-2) + 3 = -7$ and $(-2)3 = -6$, so our factorization is

$$5x^2 - 7x - 6 = (5x + 3)(x - 2).$$