MATH 111: HOMEWORK 03

BLAKE FARMAN
UNIVERSITY OF SOUTH CAROLINA

1.3

11. *Using the given set of data,*

x	y	*First difference*
0	205	
1	218	
2	231	
3	244	
4		
5		
6		

(a) Find the first differences,
(b) Is a linear model appropriate? If so, find a linear model for the data.
(c) If there is a linear model, use it to complete the table.

Solution. (a) The first differences and the missing data are given in the table below.

x	y	First difference
0	205	—
1	218	13
2	231	13
3	244	13
4	257	13
5	270	13
6	283	13

(b) Looking at the first differences, we see that it is constant and so a linear model is appropriate for the data.

(c) Looking at the table, the initial value is 205 and the constant rate of change is 13, so the linear model is given by

$$y = 13x + 205.$$

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21. *Most high-altitude hikers know that cooking takes longer at higher elevations. This is because the atmospheric pressure decreases as the elevation increases, causing water to boil at a lower temperature, and food cooks more slowly at that lower temperature. The table below gives data for the boiling point of water at different elevations.*

(a) Use first differences to show that a linear model is appropriate for the data.
(b) Find a linear model for the relation between boiling point and elevation.
(c) Use the model to predict the boiling point of water at the peak of Mount Kilimanjaro, $19,340$ *feet above sea level.*

Elevation (×1000 ft)	*Boiling point (°F)*	*First difference*
0	212.0	—
1	210.2	
2	208.4	
3	206.6	
4	204.8	
5	203.0	

Solution. (1) The first differences are given in the table below.

Elevation (×1000 ft)	Boiling point (°F)	First difference
0	212.0	—
1	210.2	-1.8
2	208.4	-1.8
3	206.6	-1.8
4	204.8	-1.8
5	203.0	-1.8

Since the first differences are constant, a linear model is appropriate.

(2) Using the initial value of 212.0 from the table and our constant rate of change, -1.8, we have the model
$$y = -1.8x + 212.0.$$

(3) When the elevation is $x = 19340/1000 = 19.34$, the boiling point will be
$$-1.8(19.34) + 212.0 = 177.188.$$

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7. *A set of ordered pairs defining a relation are given. Is the relation a function?*
$$\{(1,2),(-1,2),(3,4),(-3,5),(5,8),(-5,8)\}.$$

Solution. Yes, the relation is a function. For each (x, y) pair, the x value corresponds to exactly one y value.

9. *A set of ordered pairs defining a relation are given. Is the relation a function?*

$$\{(3,4),(4,-1),(3,5),(-1,5),(3,9),(-2,7)\}.$$

Solution. No, this relation is not a function. The value 3 corresponds to both 4 and 5.

11. *Two-variable data are given in the table below*

x	1	2	2	4	4
y	6	9	3	2	8

(a) Is the variable y a function of the variable x? If so, which is the independent variable and which is the dependent variable?

(b) Is the variable x a function of the variable y? If so, which is the independent variable and wchich is the dependent variable?

Solution. (1) No, the variable y is not a fucntion of the variable x. The x-value 4 maps to 2 and 8.

(2) Yes, the variable x is a function of y. Each y value corresponds to exactly one value of x.

13. *Two-variable data are given in the table below*

x	1	2	3	4	5	6
y	10	9	10	15	10	21

(a) Is the variable y a function of the variable x? If so, which is the independent variable and which is the dependent variable?

(b) Is the variable x a function of the variable y? If so, which is the independent variable and wchich is the dependent variable?

Solution. (1) Yes, the variable y is a function of the variable x. Each x value corresponds to exactly one value of y.

(2) No, the variable x is not a function of the variable y. The value 10 corresponds to the values 1, 3, and 5.

21. *Consider the function $w = 5\ell^2 + 3\ell^3$.*

(a) What is the independent variable and what is the dependent variable?

(b) What is the value of the dependent variable when the value of the independent variable is 2?

Solution. (1) The independent variable is ℓ and the dependent variable is w.

(2) The value of w when $\ell = 2$ is

$$5(2)^2 + 3(2)^3 = 5(4) + 3(8) = 20 + 24 = 44.$$

57. *A university music department plans to stage the opera Carmen. The fixed cost for the set, costumes, and lighting is \$5000, and they plan to charge \$15 a ticket. So if they sell x tickets, then the profit P they will make from the performance is given by the equation*

$$P = 15x - 5000.$$

(a) Show that P is a function of x.
(b) Find the net change in the profit P when the number of tickets sold increases from 100 to 200.
(c) Express x as a f unction of P.
(d) Find the net change in the number of tickets sold when the profit changes from \$0 to \$5000.

Solution. (a) The variable P is a function of x because each value of x gives exactly one P-value.
(b) The net change in P from $x = 100$ to $x = 200$ is given by

$$\begin{aligned}(15(200) - 5000) - (15(100) - 5000) &= 15(200) - 5000 - 15(100) + 5000 \\ &= 15(200) - 15(100) - 5000 + 5000 \\ &= 15(200 - 100) \\ &= 15(100) \\ &= 1500.\end{aligned}$$

(c) To express x as a function of P we solve the equation $P = 15x - 5000$ for x. Adding 5000 to both sides we obtain $P + 5000 = 15x$. Then we obtain the final answer by dividing both sides by 15,

$$x = \frac{P + 5000}{15}.$$

(d) To find the net change in the number of tickets sold when the profit changes from $P = 0$ to $P = 5000$ we use the equation

$$x = \frac{P + 5000}{15}$$

and the net change formula to obtain

$$\begin{aligned}
\left(\frac{(5000)+5000}{15}\right)-\left(\frac{(0)+5000}{15}\right) &= \frac{2(5000)}{15}-\frac{5000}{15}\\
&= \frac{2(5000)-5000}{15}\\
&= \frac{5000}{15}\\
&= \frac{1000}{3}\\
&\approx 333.
\end{aligned}$$

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9. *Let $f(x) = 2x + 1$.*

(a) What is the name of the function?
(b) What letter represents the input? What is the output?
(c) What rules does this function represent?
(d) Find $f(10)$. What does $f(10)$ represent?

Solution. (a) The name of the function is f.
(b) The letter x represents the input. The output is $f(x)$.
(c) This function represents the rule multiply x by 2, then add 1.
(d) Substituting 10 for x we have

$$f(10) = 2(10) + 1 = 20 + 1 = 21.$$

This represents the function value when $x = 10$.

13. *Express the rule "Add 2, then multiply by 5" in function notation.*

Solution. This rule is represented by the function

$$f(x) = 5 \cdot (x + 2).$$

27. *Consider the equation $y = 3x^2$.*

(a) Does the equation define a function with x as the independent variable and y as the dependent variable? If so, express the equation in function notation with x as the independent variable.

(b) Does the equation define a function with y as the independent variable and x as the dependent variable? If so, express the equation in function notation with y as the independent variable.

Solution. (a) Yes, this equation defines y as a function of x. In function notation, we write $f(x) = 3x^2$.

(b) No, this equation does not define x as a function of y. Solving for x in terms of y, we first divide both sides by 3 to obtain

$$\frac{y}{3} = x^2.$$

Next, we take the square root of both sides which gives

$$x = \pm\sqrt{\frac{y}{3}}.$$

Hence there are two values of x for each value of y (except at $y = 0$), and so this is *not* a function.

49. *Find the domain of the function*

$$h(x) = \frac{1}{x-3}.$$

Solution. The domain of the h is all the values of x *except* those that would place a zero in the denominator (because we cannot divide by zero). Solving the equation $x - 3 = 0$ by adding 3 to both sides, we have that $x = 3$ is the only value where h is not defined. Therefore our domain is all real numbers except $x = 3$, or in interval notation $(-\infty, 3) \cup (3, \infty)$.

51. *Find the domain of the function*

$$k(x) = \sqrt{x-5}.$$

Solution. Since the square root function only has real values when the input is non-negative, we have the constraint $x - 5 \geq 0$. Solving this linear inequality by adding 5 to both sides, we see that the domain is $x \geq 5$ or $[5, \infty)$ in interval notation.