MATH 111: HOMEWORK 03

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3.2

32. *If* $2500 *is invested at an interest rate of* 2.5% *per year, compounded daily, find the value of the investment after the given number of years.*

(a) 2 years,

(b) 6 years,

(c) 8 years.

Solution. The general form of the function for compounding interest is

$$I(t) = P\left(1 + \frac{r}{n}\right)^{nt}.$$

Our values are $P = 2500$, $r = 2.5/100$, $n = 365$, so our function is

$$I(t) = 2500\left(1 + \frac{\frac{2.5}{100}}{365}\right)^{365t} = 2500\left(1 + \frac{2.5}{36500}\right)^{365t} = 2500\left(\frac{365025}{365000}\right)^{365t}.$$

(a) When $t = 2$ we have

$$I(2) = 2500\left(\frac{365025}{365000}\right)^{2\cdot 365} = 2500\left(\frac{365025}{365000}\right)^{730} \approx 2628.17.$$

(b) When $t = 3$ we have

$$I(3) = 2500\left(\frac{365025}{365000}\right)^{3\cdot 365} = 2500\left(\frac{365025}{365000}\right)^{1095} \approx 2694.70.$$

(c) When $t = 6$ we have

$$I(6) = 2500\left(\frac{365025}{365000}\right)^{6\cdot 365} = 2500\left(\frac{365025}{365000}\right)^{2190} \approx 2904.57.$$

34. *If* $10,000 *is invested at an interest rate of* 10% *per year, compounded semiannually, find the value of the investment after the given number of years.*

(a) 5 years,

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(b) 10 years,

(c) 15 years.

Solution. Using the general form of the interest rate function with the values $P = 10000$, $r = 10/100$, and $n = 2$ we have the function

$$I(t) = 10000\left(1 + \frac{\frac{10}{100}}{2}\right)^{2t} = 10000\left(1 + \frac{1}{20}\right)^{2t} = 10000\left(\frac{21}{20}\right)^{2t}$$

(a) When $t = 5$ we have

$$I(5) = 10000\left(\frac{21}{20}\right)^{2\cdot 5} = 10000\left(\frac{21}{20}\right)^{10} \approx 16288.95.$$

(b) When $t = 10$ we have

$$I(10) = 10000\left(\frac{21}{20}\right)^{2\cdot 10} = 10000\left(\frac{21}{20}\right)^{20} \approx 26532.98.$$

(c) When $t = 15$ we have

$$I(15) = 10000\left(\frac{21}{20}\right)^{2\cdot 15} = 10000\left(\frac{21}{20}\right)^{30} \approx 43,219.42.$$

38. *Find the annual percentage yield for an investment that earns* 4% *each year, compounded montly.*

Solution. We are given the values $r = 4/100$ and $n = 12$. To compute the annual percentage yield, we start by computing the growth factor, a, from the interest formula

$$I(t) = P\left(1 + \frac{r}{n}\right)^{nt} = P\left(1 + \frac{4}{1200}\right)^{12t} = P\left(\frac{1204}{1200}\right)^{12t}.$$

To get the growth factor, we need to put this formula in the general form of an exponential function $I(t) = Pa^t$, which we do by using our rules for exponentiation to write

$$I(t) = P\left(\frac{1204}{1200}\right)^{12t} = P\left(\underbrace{\left(\frac{1204}{1200}\right)^{12}}_{a}\right)^{t}.$$

This gives us the growth factor

$$a = \left(\frac{1204}{1200}\right)^{12} \approx 1.0407.$$

Therefore our annual percentage yield is $a - 1 \approx 0.0407$ or 4.07%.

3.3

27. *The population of California was 29.76 million in 1990 and 33.87 million in 2000. Assume that the population grows exponentially.*

(a) Find an exponential growth model $f(t) = Ca^t$ for the population t years since 1990. *What is the annual growth rate?*

(b) Sketch a graph of the function found in part (a), and plot the points $(0, f(0))$, $(1, f(1))$, $(10, f(10))$, and $(11, f(11))$.

(c) Use the model found in part (a) to find the average rate of change from 1990 to 1991 and from 2000 to 2001. Is the average rate of change the same on each of these timer intervals?

(d) Use the model to find the percentage rate of change from 1990 to 1991 and from 2000 to 2001. Is the percentage change the same on each of these time intervals?

Solution. (a) First we assume that we have a function $f(t) = Ca^t$ that models out population. We are given two values for this function, $f(0) = 29.76$ and $f(10) = 33.87$. Using the first value, we have

$$f(0) = 29.76 = Ca^0 = C.$$

Hence our function looks like

$$f(t) = 29.76a^t.$$

It remains to find the value of a.

To do this, we use our second function value,

$$f(10) = 33.87 = 29.76a^{10}.$$

We now solve this equation for a. First we divide both sides by 29.76 and then we take the *positive* 10^{th} root of each side to obtain

$$a = \sqrt[10]{\frac{33.87}{29.76}} = \left(\frac{33.87}{29.76}\right)^{\frac{1}{10}} \approx 1.013.$$

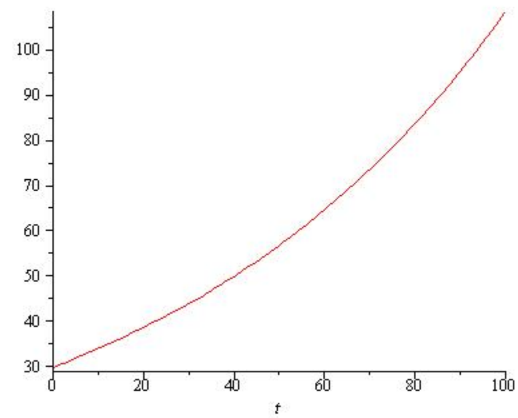
This gives us our growth factor and our growth rate, $r = a - 1 \approx 0.013$, or 1.3%. The model is

$$f(t) = 29.76\left(\frac{33.87}{29.76}\right)^{\frac{t}{10}}.$$

(b) The table below gives the approximate values of the function f at each of the specified t values

t	$f(t)$
0	29.76
1	30.15
10	33.87
11	34.31

A graph of the function is given below:



(c) The average rate of change for the period 1990 to 1991 is given by

$$\frac{f(1) - f(0)}{1 - 0} = f(1) - f(0) \approx 30.15 - 29.76 = 0.39.$$

The average rate of change for the period 2000 to 2001 is given by

$$\frac{f(11) - f(10)}{1 - 0} = f(11) - f(10) \approx 34.31 - 33.87 = 0.44.$$

This shows that the average rate of change for the exponential function is not the same over different time periods.

(d) The percentage rate of change for the period 1990 to 1991 is given by

$$\frac{f(1) - f(0)}{f(0)} \approx \frac{30.15 - 29.76}{29.76} \approx 0.013.$$

The percentage rate of change for the period 2000 to 2001 is given by

$$\frac{f(11) - f(10)}{f(10)} \approx \frac{34.31 - 33.87}{33.87} \approx 0.013.$$

The percentage rate of change for these two time periods is the same (up to precision errors in the computation), which is what we expect from exponential functions.