MATH 111: HOMEWORK 08 SOLUTIONS

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4.1

16. *Find the given logarithm.*

(a) $\log_9(1)$,
(b) $\log_9(9^8)$,
(c) $\log_9(9)$.

Solution. (a) Since $1 = 9^0$, we have $\log_9(1) = 0$.
(b) Here $\log_9(9^8) = 8$.
(c) Here $\log_9(9^1) = 1$.

18. *Find the given logarithm.*

(a) $\log_7(1)$,
(b) $\log_7(49)$,
(c) $\log_7(\frac{1}{49})$.

Solution. (a) Since $7^0 = 1$ we have $\log_7(1)$.
(b) If we write $49 = 7^2$, then we have $\log_7(49) = \log_7(7^2) = 2$.
(c) Using the laws of logarithms we have

$$\log_7\left(\frac{1}{49}\right) = \log_7(1) - \log_7(49) = 0 - 2 = -2.$$

24. *Find the given logarithms*

(a) $\log_3\left(\frac{1}{27}\right)$,
(b) $log_{10}(\sqrt{10})$,
(c) $\log_5(0.2)$.

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Solution. (a) If we write $27 = 3^3$, then by the laws of logarithms we have

$$\log_3\left(\frac{1}{27}\right) = \log_3(1) - \log_3(27) = 0 - \log_3(3^3) = -3.$$

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(b) If we write $\sqrt{10} = 10^{\frac{1}{2}}$ we have

$$log_{10}(\sqrt{10}) = \log_{10}(10^{\frac{1}{2}} = \frac{1}{2}.$$

(c) First, write $0.2 = \frac{2}{10} = \frac{1}{5}$ so that by the laws of logarithms

$$\log_5(0.2) = \log_5\left(\frac{1}{5}\right) = \log_5(1) - \log_5(5) = 0 - 1 = -1.$$

32. *Express the equation in exponential form.*

(a) $\log_3(81) = 4$,

(b) $\log_2\left(\frac{1}{8}\right)$.

Solution. (a) The equation $\log_3(81) = 4$ is true if and only if $3^4 = 81$, which is the exponential form.

(b) The equation $\log_2\left(\frac{1}{8}\right) = -3$ is true if and only if $2^{-3} = \frac{1}{8}$.

34. *Express the equation in logarithmic form.*

(a) $10^3 = 1000$,

(b) $81^{1/2} = 9$.

Solution. (a) The equation $10^3 = 1000$ tells us that $\log_{10}(1000) = 3$.

(b) The equation $81^{1/2} = 9$ tells us that $\log_{81}(9) = \frac{1}{2}$.

4.2

10. *Evaluate the given expression.*

(a) $\log_{10}(4) + log_{10}(25)$,

(b) $\log_2(160) - \log_2 5$,

(c) $-\frac{1}{2}\log_2(64)$.

Solution. (a) Using the law for multiplication we have

$$\log_{10}(4) + log_{10}(25) = \log_{10}(4 \cdot 25) = \log_{10}(100) = \log_{10}(10^2) = 2.$$

(b) Using the law for division we have

$$\log_2(160) - \log_2 5 = \log_2\left(\frac{160}{5}\right) = \log_2(32) = \log_2(2^5) = 5.$$

(c) Using the law for exponents we have

$$-\frac{1}{2}\log_2(64) = -\log_2(64^{1/2}) = -\log_2((2^6)^{1/2}) = -\log_2(2^{6/2}) = -\log_2(2^3) = -3.$$

12. *Use the laws of logarithms to expand the given expression.*

(a) $\log_5\left(\frac{x}{2}\right)$,

(b) $\log_3(x\sqrt{y})$.

Solution. (a) Using the law for division we have

$$\log_5\left(\frac{x}{2}\right) = \log_5(x) - \log_5(2).$$

(b) Using the law for products and then the law for exponents we have

$$\log_3(x\sqrt{y}) = \log_3(xy^{1/2}) = \log_3(x) + \log_3(y^{1/2}) = \log_3(x) + \frac{\log_3(y)}{2}.$$

14. *Use the laws of logarithms to expand the given expression.*

(a) $\log_3(5a)$,

(b) $\log_5\left(\frac{2a}{b}\right)$.

Solution. (a) Using the law for products we have

$$\log_3(5a) = \log_3(5) + \log_3(a).$$

(b) Using the law for division, then the law for products we have

$$\log_5\left(\frac{2a}{b}\right) = \log_5(2a) - \log_5(b) = \log_5(2) + \log_5(a) - \log_5(b).$$

16. *Use the laws of logarithms to expand the given expression.*

(a) $\log_{10}(w^2 z)^{10}$

(b) $\log_7\left(\frac{\sqrt[3]{wz}}{x}\right)$

Solution. Using the laws of logarithms we have

(a)

$$\log_{10}(w^2 z)^{10} = (\log_{10}(w^2) + \log_{10}(z))^{10} = (2\log_{10}(w) + \log_{10}(z))^{10}.$$

(b)

$$\begin{aligned}
\log_7\left(\frac{\sqrt[3]{wz}}{x}\right) &= \log_7\left(\frac{(wz)^{1/3}}{x}\right) \\
&= \log_7((wz)^{1/3}) - \log_7(x) \\
&= \frac{log_7(wz)}{3} - \log_7(x) \\
&= \frac{log_7(w) + \log_7(z)}{3} - \log_7(x) \\
&= \frac{log_7(w)}{3} + \frac{\log_7(z)}{3} - \log_7(x)
\end{aligned}$$

20. *Use the laws of logarithms to combine the given expression.*
(a) $4\log_2(x) - \frac{1}{3}\log_2(x^2+1)$.
(b) $\log_{10}(5) + 2\log_{10}(x) + 3\log_{10}(x^2+5)$.

Solution. Using the laws of logarithms we have

(a)

$$\begin{aligned}
4\log_2(x) - \frac{1}{3}\log_2(x^2+1) &= \log_2(x^4) - \log_2(\sqrt[3]{x^2+1}) \\
&= \log_2(\frac{x^4}{\sqrt[3]{x^2+1}})
\end{aligned}$$

(b)

$$\begin{aligned}
\log_{10}(5) + 2\log_{10}(x) + 3\log_{10}(x^2+5) &= \log_{10}(5) + \log_{10}(x^2) + \log_{10}((x^2+5)^3) \\
&= \log_{10}(5x^2(x^2+5)^3) \\
&= \log_{10}(5x^2(x^6 + 15x^4 + 75x^2 + 125)) \\
&= \log_{10}(5x^8 + 75x^6 + 375x^4 + 625x^2).
\end{aligned}$$

22. *Use the laws of logarithms to combine the given expression.*
(a) $2\log_8(x+1) + 2\log_8(x-1)$
(b) $\log_5(x^2-1) - \log_5(x-1)$.

Solution. Using the laws of logarithms we have

(a)

$$\begin{aligned}
2\log_8(x+1) + 2\log_8(x-1) &= 2(\log_8(x+1) + \log_8(x-1)) \\
&= 2(\log_8((x+1)(x-1))) \\
&= 2(\log_8(x^2-1)) \\
&= \log_8((x^2-1)^2) \\
&= \log_8(x^4 - 2x^2 + 1).
\end{aligned}$$

(b)

$$\log_5(x^2-1) - \log_5(x-1) = \log_5\left(\frac{x^2-1}{x-1}\right) = \log_5\left(\frac{(x+1)(x-1)}{x-1}\right) = \log_5(x+1).$$

34. *Use the change of base formula and a calculator to evaluate the logarithm.*
(a) $\log_3(16)$,
(b) $\log_6(92)$.

Solution. Using the change of base formula and a calculator we have
(a)

$$\log_3(16) = \frac{\log_{10}(16)}{\log_{10}(3)} \approx 2.523719.$$

(b)

$$\log_6(92) = \frac{\log_{10}(92)}{\log_{10}(6)} \approx 2.523658.$$

36. *Use the change of base formula and a calculator to evaluate the logarithm.*
(a) $\log_4(125)$,
(b) $\log_{12}(2.5)$.

Solution. Using the change of base formula and a calculator we have
(a)

$$\log_4(125) = \frac{\log_{10}(125)}{\log_{10}(4)} \approx 3.482892.$$

(b)

$$\log_{12}(2.5) = \frac{\log_{10}(2.5)}{\log_{10}(12)} \approx 0.368743.$$