MATH 116: EXAM 02

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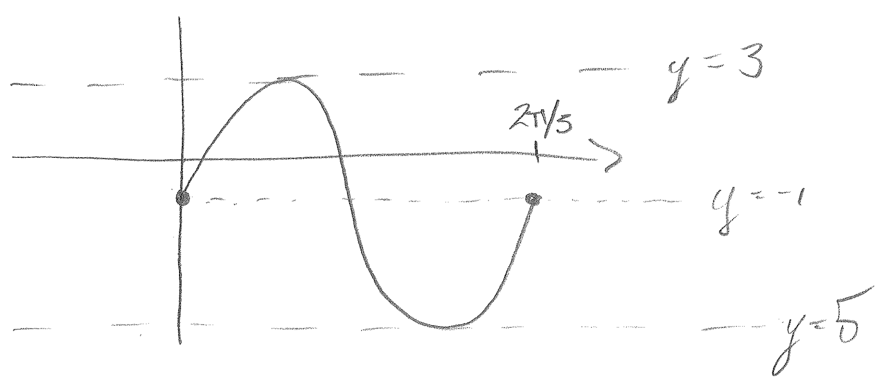
Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. Unless otherwise stated, all supporting work is required. You may *not* use any calculators.

Name: Answer Key

1 (20 Points). *Find the period, frequency, and amplitude of* $y = 4\sin(3x) - 1$, *then graph one period.*

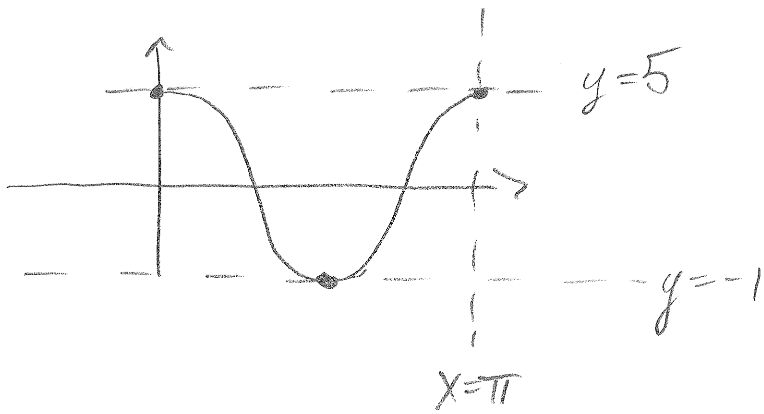
The period is $2\pi/3$, the frequency is $3/2\pi$, the amplitude is 4.



Date: December 2, 2013.

2 (20 Points). *Find the period, frequency, and amplitude of* $y = 3\cos(2x) + 2$, *then graph one period.*

The period is $\frac{2\pi}{2} = \pi$, the frequency is $1/\pi$, and the amplitude is 3.



3 (20 Points). *Let* $f(x) = x^2 - 2x$ *and* $g(x) = \sqrt{x}$.

(a) Compute $(f \circ g)(x)$.

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 - 2(\sqrt{x})$
$= x - 2\sqrt{x}.$

(b) Compute $(g \circ f)(x)$.

$(g \circ f)(x) = g(f(x)) = g(x^2 - 2x) = \sqrt{x^2 - 2x} = \sqrt{x(x-2)}$
$= \sqrt{x}\sqrt{x-2}.$

4 (20 Points). *Determine whether* $g(x) = \sqrt[3]{5x+1}$ *is invertible. If it is, then compute the inverse. Otherwise, explain why it does not have an inverse.*

This function is invertible.

$y = \sqrt[3]{5x+1}$

$\Rightarrow y^3 = 5x+1$

$\Rightarrow y^3 - 1 = 5x$

$\Rightarrow \cancel{y} \frac{y^3-1}{5} = x$

$\Rightarrow g^{-1}(x) = \frac{x^3-1}{5}.$

5 (20 Points). *Solve the following equations for x.*

(a)

$$2\log_2(\sqrt{x+2}) - \log_2\left(\frac{1}{x-2}\right) = 5$$

Use the log rules $\log_a(x^r) = r\log_a(x)$ to reduce:

$$2\log_2(\sqrt{x+2}) = \log_2((\sqrt{x+2})^2) = \log_2(x+2)$$

and

$$-\log_2\left(\frac{1}{x-2}\right) = \log_2\left(\left(\frac{1}{x-2}\right)^{-1}\right) = \log_2(x-2).$$

Then use the log rule $\log_a(x) + \log_a(y) = \log_a(xy)$:

$$5 = 2\log_2(\sqrt{x+2}) - \log_2\left(\frac{1}{x-2}\right) = \log_2(x+2) + \log_2(x-2)$$
$$= \log_2((x+2)(x-2))$$
$$= \log_2(x^2-4)$$

$$\Rightarrow 2^5 = x^2 - 4 \Rightarrow x^2 = 2^5 + 4 = 32 + 4 = 36 \Rightarrow x = \sqrt{36} = 6.$$

(b)

$$2^{-4x} = 16 \cdot 2^{x^2}$$

Rewrite 16 as 2^4, then

$$2^{-4x} = 2^4 \cdot 2^{x^2} = 2^{x^2+4}$$

$$\Rightarrow \log_2(2^{-4x}) = \log_2(2^{x^2+4})$$

$$\Rightarrow -4x = x^2 + 4$$

$$\Rightarrow x^2 + 4x + 4 = 0$$

$$\Rightarrow (x+2)^2 = 0$$

$$\Rightarrow x = -2.$$