1.1

13) $$\frac{\left(\frac{xy}{(x+y)}\right)}{\left(\frac{x^2y}{(x+y)^3}\right)} = \frac{xy}{(x+y)}\left(\frac{(x+y)^3}{x^2y}\right) = \frac{(x+y)^3}{(x+y)}\,\frac{(xy)}{(x^2y)} = \frac{(x+y)^2}{x}. \blacksquare$$

14) $$\frac{\left(\frac{xy}{(x-y)}\right)}{\left(\frac{x^2}{y}\right)\left(\frac{y^3}{x}\right)} = \left(\frac{xy}{(x-y)}\right)\frac{xy}{x^2y^3} = \frac{1}{xy-y^2}. \blacksquare$$

1.2

16) $$\frac{\left(\frac{1}{x}-\frac{1}{y}\right)}{\left(\frac{1}{x}+\frac{1}{y}\right)} = \frac{\left(\frac{y-x}{xy}\right)}{\left(\frac{y+x}{xy}\right)} = \left(\frac{y-x}{xy}\right)\left(\frac{xy}{y+x}\right) = \frac{y-x}{y+x}. \blacksquare$$

21) $$\frac{4yz}{x^2} - \frac{2z}{xy^2} + \frac{1}{xyz}$$

The common denominator here is x^2y^2z, the least common multiple of x^2, xy^2, and xyz, so

$$\frac{4yz}{x^2} - \frac{2z}{xy^2} + \frac{1}{xyz} = \frac{4yz(y^2z) - 2z(xz) + xy}{x^2y^2z}$$
$$= \frac{4y^3z^2 - 2xz^2 + xy}{x^2y^2z}. \blacksquare$$

1.3

7) $$2x(y-3) - y(x+xy) + 2y(x+1) = 2xy - 6x - xy - xy^2 + 2xy + 2y$$
$$= 2xy - xy + 2xy - 6x - xy^2 + 2y$$
$$= 3xy - 6x - xy^2 + 2y. \blacksquare$$

8) $$x(y+z) - z(x+y) + 2y(x-z) - x(3y-2z) = xy + xz - xz - yz + 2xy - 2yz - 3xy + 2xz$$
$$= xy + 2xy + xz - xz - yz - 2yz - 3xy + 2xz$$
$$= 2xz - 3yz. \blacksquare$$

1.4

10. Write $(x^{-2}+y^{-2})^2 = \left(\frac{1}{x^2}+\frac{1}{y^2}\right)^2$ and $x^{-4}+y^{-4} = \frac{1}{x^4}+\frac{1}{y^4}$.

Take $x=1$ and $y=1$, then

$$\left(\frac{1}{(1)^2}+\frac{1}{(1)^2}\right)^2 = \left(\frac{1}{1}+\frac{1}{1}\right)^2 = (1+1)^2 = 2^2 = 4,$$

and

$$\frac{1}{(1)^4}+\frac{1}{(1)^4} = \frac{1}{1}+\frac{1}{1} = 1+1 = 2 \neq 4.$$

Therefore $(x^{-2}+y^{-2})^2 \neq (x^{-4}+y^{-4})$. $\blacksquare$

14) $\left(\frac{x^4y^2}{x^{-3}}\right) \div \left(\frac{x^3y^{-2}}{y^5}\right) = \left(\frac{x^4y^2}{x^{-3}}\right)\left(\frac{y^5}{x^3y^{-2}}\right)$

$$= (x^3)(x^4)(y^2)\left(\frac{(y^5)(y^2)}{x^3}\right)$$

$$= (x^4)(y^2)(y^5)(y^2)$$

$$= x^4y^9. \blacksquare$$

1.5

14) $\left(\frac{25}{16}\right)^{-3/2} = \left(\frac{16}{25}\right)^{3/2} = \left(\sqrt{\frac{16}{25}}\right)^3 = \left(\frac{\sqrt{16}}{\sqrt{25}}\right)^3 = \left(\frac{4}{5}\right)^3 = \frac{64}{125}. \blacksquare$

30) If $x^2+y^2=25$, can we conclude that $x+y=5$? Why or why not?

First we observe that both x^2 and y^2 are positive, so we have a small set of squares to consider. Namely, the only possibilities are

$$25 = 5^2+0^2 = 0^2+5^2$$

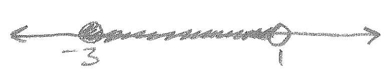
and

$$25 = 16+9 = 4^2+3^2.$$

The latter is a counter-example: $4+3=7\neq 5$. Therefore we may not conclude that if $x^2+y^2=25$, then $x+y=5$. $\blacksquare$

1.8

1.

Inequality	Number Line	Interval
a) $-1 \le x \le 3$		$[-1, 3]$
b) $-1 < x \le 3$		$(-1, 3]$
c) $-3 \le x < 1$		$[-3, 1)$
d) $-3 \le x \le 4$		$[-3, 4]$.

3)

Interval	Inequality
a) $(3,7)$	$3 < x < 7$
b) $(-4,-1]$	$-4 < x \leq -1$
c) $(-\infty, 19]$	$x \leq 19$
d) $[-2, 10)$	$2 \leq x < 10$
e) $[-2, -1]$	$-2 \leq x \leq -1$. ∎

5) Simplify if possible.

a) $(-\infty,5) \cap [3,\infty)$ is the set of numbers satisfying $x<5$ and $3 \leq x$, so this is the set of numbers between 3 and 5, excluding 5, which we may write as

$$3 \leq x < 5 \text{ or } [3,5). \blacksquare$$

b) $(-\infty,5) \cup [3,\infty)$ is the set of numbers that are either strictly smaller than 5 or at least as large as 3. Since the second set includes 5 and every number satisfies one of $3 \leq x$ or $x<5$, this is the entire set of real numbers. $(-\infty,\infty)$ or $\mathbb{R}$. ∎

c) $(-\infty,-2) \cap [-2,\infty)$ is the empty set. Since 2 is not in $(-\infty,-2)$ it is not in the intersection, and this is the only possible point where these sets could intersect. ∎

d) $(-\infty,\infty) \cap [4,7] = [4,7]$. This is because $[4,7]$ is contained in $(-\infty,\infty)$. ∎

e) $[3,5] \cap (10,\infty) = \emptyset$. This is because there is no overlap between the two sets. ∎

f) $(-\infty,5] \cap [5,\infty) = \{5\}$. This is because the only point where these two intervals overlap is at 5. ∎