MATH 116
EXAM 01

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.** You may *not* use any calculators.

Name: Answer Key

Problem	Points Earned	Points Possible
1		6
2		1
3		1
4		2
5		18
6		18
7		18
8		18
9		18
Total		100

Date: November 9, 2015.

1. Definitions

1 (6 Points). *Let a, b be non-zero real numbers and m, n rational numbers. Fill in the blanks*

(i) $a^0 = \underline{1}$,

(ii) $\left(\frac{a}{b}\right)^{-n} = \underline{b^n / a^n}$.

(iii) $a^m \cdot a^n = \underline{a^{m+n}}$

(iv) $\frac{a^m}{a^n} = \underline{a^{m-n}}$

(v) $(a \cdot b)^n = \underline{a^n b^n}$

(vi) $\left(\frac{a}{b}\right)^n = \underline{a^n / b^n}$

2 (1 Points). *State the Quadratic Formula.*

The solutions to $ax^2+bx+c=0$ are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

3 (1 Points). *Fill in the blanks:*

To make $x^2 + bx$ a perfect square, add and subtract $\boxed{\frac{b}{2}}^2$. *This gives*

$$x^2 + bx + \boxed{\frac{b}{2}}^2 - \boxed{\frac{b}{2}}^2 = (x + \boxed{\frac{b}{2}})^2 - \boxed{\frac{b}{2}}^2.$$

4 (2 Points). *(a) State the Point-Slope form of a line passing through the point (x_0, y_0) with slope m.*

$y - y_0 = m(x - x_0)$

(b) State the Slope-Intercept form of a line with slope m and y-intercept b.

$y = mx + b$

2. Exercises

5 (18 Points). *In the following problems, use the given information to find the equation of the line in Slope-Intercept Form and then graph the lines.*

(a) *The line passing through the point* $(1, 1)$ *and parallel to the line* $3y - 6x = 1$.

$3y - 6x = 1 \Rightarrow 3y = 6x + 1$

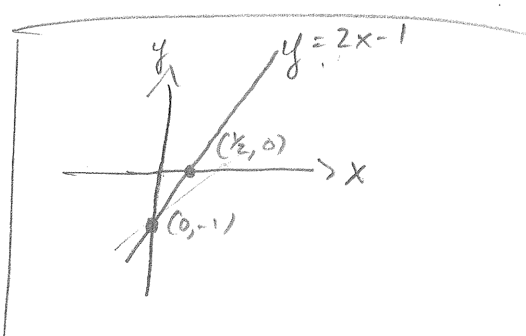
$\Rightarrow y = 2x + \frac{1}{3}$

has slope $m = 2$. The line parallel to $3y - 6x = 1$ passing through $(1,1)$ is

$y - 1 = 2(x-1)$ (Point-Slope)

$\Rightarrow y = 2x - 2 + 1$

$\Rightarrow y = 2x - 1$ (Slope-Intercept)



(b) *The line passing through the point* $(2, 2)$ *and perpendicular to the line* $3y - 6x = 1$.

A line perpendicular to $3y - 6x = 1$ has slope m satisfying

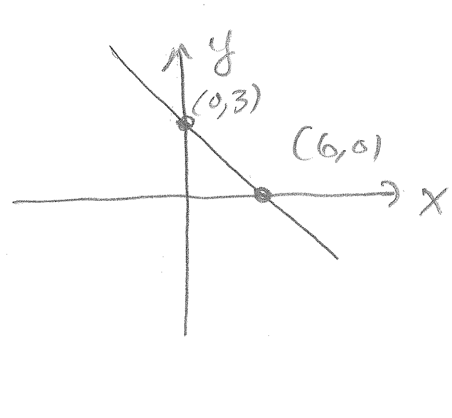
$m \cdot 2 = -1 \Rightarrow m = -\frac{1}{2}$.

The line with slope $-\frac{1}{2}$ passing through $(2,2)$ is

$y - 2 = -\frac{1}{2}(x-2)$ (Point-Slope)

$\Rightarrow y = -\frac{1}{2}x + 1 + 2$

$\Rightarrow y = -\frac{1}{2}x + 3$ (Slope-Intercept)



6 (18 Points). *Find all solutions, real and complex, to the equation*

$$x^2 + 2x + 3 = 0.$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-12}}{2}$$

$$= \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm i\sqrt{8}}{2} = \frac{-2 \pm i\sqrt{4}\sqrt{2}}{2} = \frac{-2 \pm i2\sqrt{2}}{2} = -1 \pm i\sqrt{2}.$$

7 (18 Points). *(a) Complete the square for the function* $f(x) = x^2 + 5x + 6$.

$$f(x) = x^2 + 5x + (5/2)^2 - (5/2)^2 + 6$$

$$= (x + 5/2)^2 - \frac{25}{4} + \frac{24}{4}$$

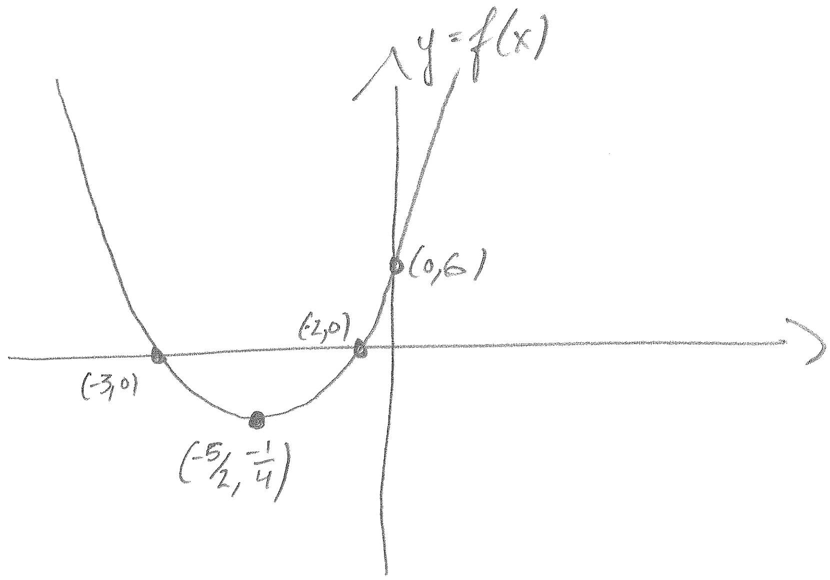
$$= (x + 5/2)^2 - 1/4.$$

(b) Solve $f(x) = 0$.

$$0 = f(x) = x^2 + 5x + 6 = (x+3)(x+2)$$

$$\Rightarrow x = -3 \text{ or } x = 2.$$

(c) Use the information from parts (a) and (b) to sketch a graph of $f(x)$. *Label the* y*-intercept, any* x*-intercept(s), and the vertex.*



8 (18 Points). *Find all solutions, both real and complex, of the equation*

$$x^4 - 18x^2 + 81 = 0.$$

Let $y = x^2$. Then

$y^2 - 18y + 81 = x^4 - 18x^2 + 81$

$y^2 - 18y + 81 = y^2 - 2(9)y + 9^2 = (y-9)^2 = 0$

has the solution $y = 9$. Since $y = x^2 = 9$, we see that $x = \pm 3$.

9 (18 Points). *Find the simultaneous solutions to the following system*

$$\begin{cases} y = x^2 - 7x + 11, \\ y = -x + 2. \end{cases}$$

It suffices to solve

$x^2 - 7x + 11 = -x + 2$

for x.

$x^2 - 7x + 11 = -x + 2$

$\Rightarrow x^2 - 7x + x + 11 - 2 = 0$

$\Rightarrow x^2 - 6x + 9 = 0$

$x^2 - 6x + 9 = x^2 - 2(3)x + 3^2 = (x-3)^2$

so $x = 3$ is the only solution. Therefore the only point of intersection is when $x = 3$ and $y = -(3) + 2 = -1$, or as a pair

$(3, -1)$.