MATH 116
EXAM 02

BLAKE FARMAN
UNIVERSITY OF SOUTH CAROLINA

Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.** You may *not* use any calculators.

Name: Answer Key

Problem	Points Earned	Points Possible
1		20
2		20
3		20
4		20
5		20
Total		100

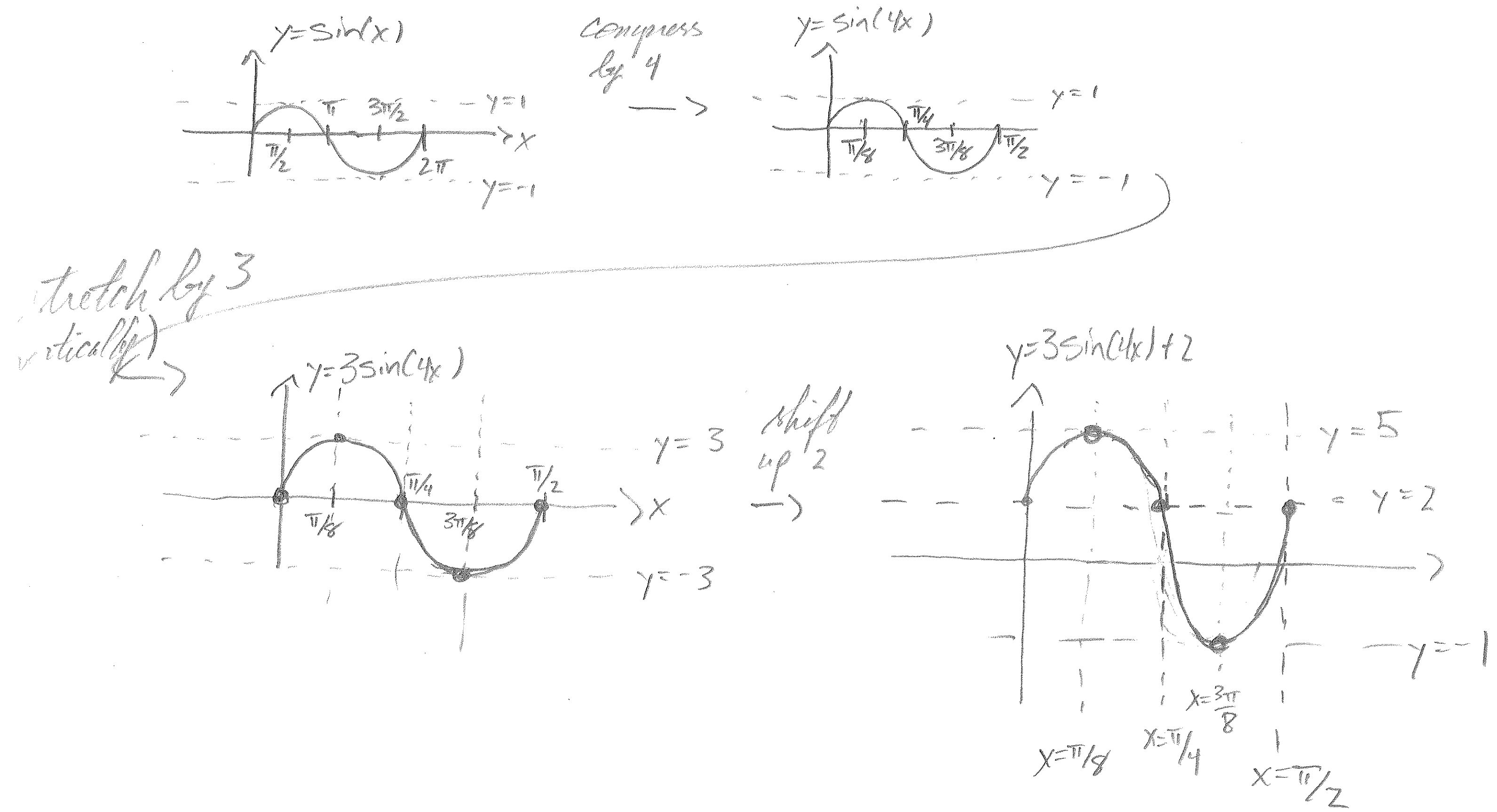
Date: November 24, 2015.

1 (20 Points). *Find the period, frequency, and amplitude of* $y = 3\sin(4x) + 2$, *then graph one period.*

Period: $2\pi/4 = \pi/2$

Frequency: $2/\pi$

Amplitude: 3

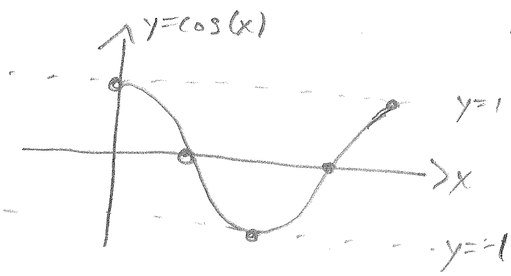


2 (20 Points). *Find the period, frequency, and amplitude of $y = 2\cos(3x) - 1$, then graph one period.*

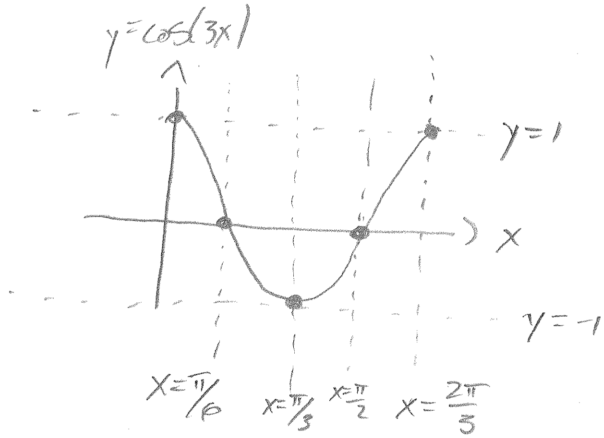
Period: $2\pi/3$

Frequency: $3/2\pi$

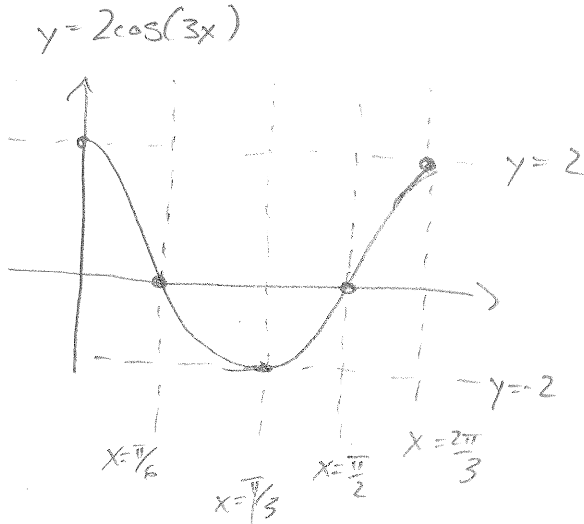
Amplitude: 2



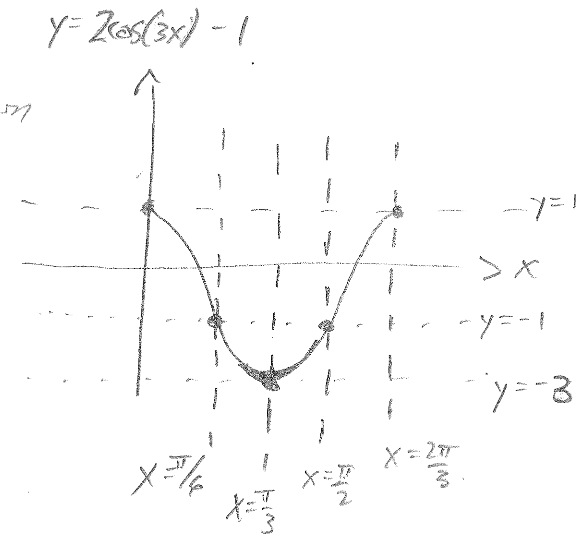
horiz. compress by 3 →



vert. stretch by 2 →



shift down 1 →



3 (20 Points). *Let* $f(x) = x^2 + 4$ *and* $g(x) = \sqrt{x}$.

(a) Compute $(f \circ g)(x)$.

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 4 = x^2 + 4$

(b) Compute $(g \circ f)(x)$.

$(g \circ f)(x) = g(f(x)) = g(x^2+4) = \sqrt{x^2+4}$.

4 (20 Points). *Determine whether $g(x) = \sqrt[3]{1-x^3}$ is invertible. If it is, then compute the inverse. Otherwise, explain why it does not have an inverse.*

$$y = \sqrt[3]{1-x^3} = (1-x^3)^{1/3}$$

$$\Rightarrow y^3 = \left((1-x^3)^{1/3}\right)^3 = 1-x^3$$

$$\Rightarrow y^3 + x^3 = 1$$

$$\Rightarrow x^3 = 1-y^3$$

$$\Rightarrow x = \sqrt[3]{1-y^3}$$

So $g(x)$ is invertible, and $g^{-1}(x) = g(x)$.

Check: $(g \circ g)(x) = g(g(x))$

$$= g\left(\sqrt[3]{1-x^3}\right)$$

$$= \sqrt[3]{1-\left(\sqrt[3]{1-x^3}\right)^3}$$

$$= \sqrt[3]{1-(1-x^3)}$$

$$= \sqrt[3]{1-1+x^3}$$

$$= \sqrt{x^3}$$

$$= x.$$

as desired.

5 (20 Points). *Solve the following equations for x.*

(a)

$$3\log_3(\sqrt[3]{x+3}) - \log_3\left(\frac{1}{x-3}\right) = 3$$

$$\begin{aligned} 3 &= 3\log_3(\sqrt[3]{x+3}) - \log_3\left(\tfrac{1}{x-3}\right) \\ &= \log_3\left((\sqrt[3]{x+3})^3\right) - \log_3(1/x-3) \\ &= \log_3(x+3) - \log_3(1/x-3) \\ &= \log_3\left(x+3/(1/x-3)\right) \\ &= \log_3\left((x+3)(x-3)\right) \\ &= \log_3(x^2-9) \end{aligned}$$

$\Rightarrow 3^3 = 3^{\log_3(x^2-9)}$

$\Rightarrow 27 = x^2-9$

$\Rightarrow x^2 = 27+9 = 36$

$\Rightarrow \boxed{x=6}$

$x=-6$ is an extraneous solution.

(b)

$$3^{-4x} = 9\cdot 3^{2x^2}$$

$3^{-4x} = 9 \cdot 3^{2x^2}$

$\Rightarrow \log_3(3^{-4x}) = \log_3(9\cdot 3^{2x^2})$

$\Rightarrow -4x = \log_3(9) + \log_3(3^{2x^2})$

$\Rightarrow -4x = 2 + 2x^2$

$\Rightarrow 2x^2 + 4x + 2 = 0$

$\Rightarrow 2(x^2+2x+1) = 0$

$\Rightarrow x^2 + 2x + 1 = 0$

$\Rightarrow (x+1)^2 = 0$

$\Rightarrow \boxed{x=-1.}$