

2.1

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$$\begin{aligned} 1) a) f(x) &= x^2 - 6x + 15 \\ &= (x^2 - 2(3)x + 9) + 6 \\ &= (x-3)^2 + 6. \quad \blacksquare \end{aligned}$$

$$\begin{aligned} b) h(y) &= y^2 + 5y \\ &= (y^2 + 2(\frac{5}{2})y + (\frac{5}{2})^2) - (\frac{5}{2})^2 \\ &= (y + \frac{5}{2})^2 - \frac{25}{4}. \quad \blacksquare \end{aligned}$$

$$\begin{aligned} c) g(s) &= s^2 + 2s - 8 \\ &= (s^2 + 2s + 1) - 9 \\ &= (s+1)^2 - 9. \quad \blacksquare \end{aligned}$$

$$\begin{aligned} d) k(x) &= 2x^2 - 2x + 5 \\ &= 2(x^2 - 2(\frac{1}{2})x + (\frac{1}{2})^2) - 2(\frac{1}{2})^2 + 5 \\ &= 2(x - \frac{1}{2})^2 - \frac{1}{2} + \frac{10}{2} \\ &= 2(x - \frac{1}{2})^2 + \frac{9}{2}. \quad \blacksquare \end{aligned}$$

$$\begin{aligned} e) f(x) &= 3x^2 - 7x + 1 \\ &= 3(x^2 - 2(\frac{7}{6})x + (\frac{7}{6})^2) - 3(\frac{7}{6})^2 + 1 \\ &= 3(x - \frac{7}{6})^2 - 3(\frac{49}{36}) + 1 \\ &= 3(x - \frac{7}{6})^2 - \frac{49}{12} + \frac{12}{12} \\ &= 3(x - \frac{7}{6})^2 - \frac{37}{12}. \quad \blacksquare \end{aligned}$$

$$\begin{aligned} f) w(x) &= \pi x^2 + 2x \\ &= \pi(x^2 + 2(\frac{1}{\pi})x + (\frac{1}{\pi})^2) - \pi(\frac{1}{\pi})^2 \\ &= \pi(x + \frac{1}{\pi})^2 - \frac{1}{\pi}. \quad \blacksquare \end{aligned}$$

$$\begin{aligned} 2) a) f(x) &= x^2 - 8x + 12 \\ &= (x^2 - 2(4)x + 16) - 4 \\ &= (x-4)^2 - 4. \quad \blacksquare \end{aligned}$$

$$\begin{aligned} b) h(y) &= y^2 + 14y \\ &= (y^2 + 2(7)y + 7^2) - 7^2 \\ &= (y+7)^2 - 49. \quad \blacksquare \end{aligned}$$

$$\begin{aligned}
 c) \quad g(s) &= s^2 + 3s - 6 \\
 &= \left(s^2 + 2\left(\frac{3}{2}\right)s + \left(\frac{3}{2}\right)^2\right) - \left(\frac{3}{2}\right)^2 - 6 \\
 &= \left(s + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{24}{4} \\
 &= \left(s + \frac{3}{2}\right)^2 - \frac{33}{4}. \quad \blacksquare
 \end{aligned}$$

$$\begin{aligned}
 d) \quad h(x) &= 4x^2 - 8x + 3 \\
 &= 4(x^2 - 2(1)x + 1) - 4(1) + 3 \\
 &= 4(x-1)^2 - 1. \quad \blacksquare
 \end{aligned}$$

$$5) \quad a) \quad x^2 + 3x + 2y^2 - 8y = 0$$

$$\Rightarrow \left(x^2 + 2\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2\right) + 2\left(y^2 - 2(2)y + 2^2\right) = \frac{9}{4} + 2(2)^2 = \frac{9}{4} + \frac{32}{4} = \frac{41}{4}$$

$$\Rightarrow \left(x + \frac{3}{2}\right)^2 + 2(y-2)^2 = \frac{41}{4}. \quad \blacksquare$$

$$b) \quad 3x^2 + 6x - 2y^2 - 8y = -11$$

$$\Rightarrow 3\left(x^2 + 2x + 1\right) - 2\left(y^2 + 2(2)y + 2^2\right) = -11 + 3 - 8 = -16$$

$$\Rightarrow 3(x+1)^2 - 2(y+2)^2 = -16. \quad \blacksquare$$

$$c) \quad -x^2 + 4x + y^2 - 16y = 40$$

$$\Rightarrow -\left(x^2 - 2(2)x + 4\right) + \left(y^2 - 2(8)y + 8^2\right) = 40 - 4 + 64 = 100$$

$$\Rightarrow -(x-2)^2 + (y-8)^2 = 100. \quad \blacksquare$$

$$d) \quad -9x^2 + 36x - 4y^2 - 8y = 0$$

$$\Rightarrow -9\left(x^2 - 2(2)x + 4\right) - 4\left(y^2 + 2(1)y + 1\right) = -36 - 4 = -40$$

$$\Rightarrow -9(x-2)^2 + 4(y+1)^2 = 40. \quad \blacksquare$$

$$e) \quad x^2 + y^2 - 6x + 10y + 34 = 0$$

$$\Rightarrow \left(x^2 - 2(3)x + 9\right) + \left(y^2 + 2(5)y + 25\right) - 9 - 25 + 34 = 0$$

$$\Rightarrow (x-3)^2 + (y+5)^2 = 0. \quad \blacksquare$$

Rmk: This is degenerate because it has radius 0.

It consists only of the point (3, -5).

$$7) a) x^2 + y^2 - 4x - 2y = 11$$

$$\Rightarrow (x^2 - 2(2)x + 4) + (y^2 - 2(1)y + 1) = 11 + 4 + 1$$

$$\Rightarrow (x-2)^2 + (y-1)^2 = 16.$$

Center: (2, 1),

Radius:  $\sqrt{16} = 4$ . ■

$$b) x^2 + y^2 - 6x + 4y - \pi^2 + 13 = 0$$

$$\Rightarrow (x^2 - 2(3)x + 9) + (y^2 + 2(2)y + 4) - \pi^2 + 13 - 9 - 4 = 0$$

$$\Rightarrow (x-3)^2 + (y+2)^2 = \pi^2$$

Center: (3, -2)

Radius:  $\pi$ . ■

$$c) 2x^2 + 2y^2 + 4x + 8y - 20 = 0$$

$$\Rightarrow x^2 + y^2 + 2x + 4y = 10$$

$$\Rightarrow (x^2 + 2x + 1) + (y^2 + 2(2)y + 4) = 10 + 1 + 4 = 15$$

$$\Rightarrow (x+1)^2 + (y+2)^2 = 15.$$

Center: (-1, -2)

Radius:  $\sqrt{15}$ . ■

$$8) a) x^2 + y^2 - 6x - 8y = 0$$

$$\Rightarrow (x^2 - 2(3)x + 9) + (y^2 - 2(4)y + 16) = 9 + 16 = 25$$

$$\Rightarrow (x-3)^2 + (y-4)^2 = 25$$

Center: (3, 4)

Radius: 5. ■

$$b) x^2 + y^2 - 10x + 12y + 12 = 0$$

$$\Rightarrow (x^2 - 2(5)x + 25) + (y^2 + 2(6)y + 36) = -12 + 25 + 36 = 49$$

$$\Rightarrow (x-5)^2 + (y+6)^2 = 49$$

Center: (5, -6)

Radius: 7. ■