3.1

5) $\frac{4}{3}z - 1 = \frac{1}{10}$

$\Rightarrow \frac{4}{3}z = \frac{1}{10} + 1 = \frac{1}{10} + \frac{10}{10} = \frac{11}{10}$

$\Rightarrow z = \frac{3}{4}\left(\frac{11}{10}\right) = \frac{33}{40}$. ■

12) $2z^2x - z^3 = 1$

$\Rightarrow 2z^2x = 1 + z^3$

$\Rightarrow x = \frac{1+z^3}{2z^2}$.

3.2

3) $x^2 + 6x + 9 = 0$

$\Rightarrow (x+3)^2 = 0$

$\Rightarrow x = -3$. ■

7) $y^2 - 2y + 2 = 0$

$\Rightarrow y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$

$= \frac{2 \pm \sqrt{4-8}}{2}$

$= \frac{2 \pm \sqrt{-4}}{2}$

$= \frac{2 \pm i2}{2}$

$= 1 \pm i$. ■

3.3

17) $\sqrt{x} - 3 = 5 - \sqrt{x}$

$\Rightarrow \sqrt{x} + \sqrt{x} = 5 + 3$

$\Rightarrow 2\sqrt{x} = 8$

$\Rightarrow \sqrt{x} = 4$

$\Rightarrow x = 16$. ■

22. Find all real solutions to $x^{2/5} - 3x^{1/5} + 2 = 0$.

Write $x^{2/5} - 3x^{1/5} + 2 = 0$ as

$(x^{1/5})^2 - 3(x^{1/5}) + 2 = 0$

$\Rightarrow (x^{1/5} - 2)(x^{1/5} - 1) = 0$

$\Rightarrow x^{1/5} = 2$ or $x^{1/5} = 1$.

$\Rightarrow x = 32$ or $x = 1$. ■

4.2

6) Find the equation of the line of slope $2/3$ through the point $(2,7)$.

$y-7 = 2/3(x-2)$ (point-slope)

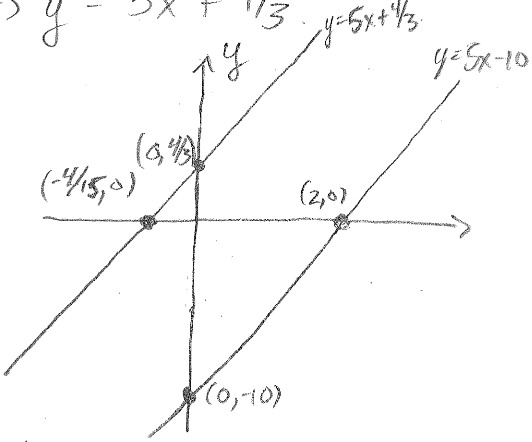
or $y = 2/3x - 2/3(2) + 7$
$= 2/3x - 4/3 + 21/3$
$= 2/3x + 17/3$ (slope-intercept). ■

15) Find the equation of the line through the point $(0, 4/3)$ parallel to the line $y = 5x-10$. Graph both lines.

Since the line we wish to find is parallel to $y=5x-10$, we use the slope $m=5$. Hence the line is

$y - 4/3 = 5(x-0) = 5x$

$\Rightarrow y = 5x + 4/3$.



17) Find the equation of the line through the point $(\frac{1}{2}, -3)$ perpendicular to the line $y = -1/7x - 5$. Graph both lines.

Since the lines are perpendicular, we want the slope of the line to be the number m such that

$m(\frac{-1}{7}) = -1$

$\Rightarrow m = (-7)(-1)$

$\Rightarrow m = 7$.

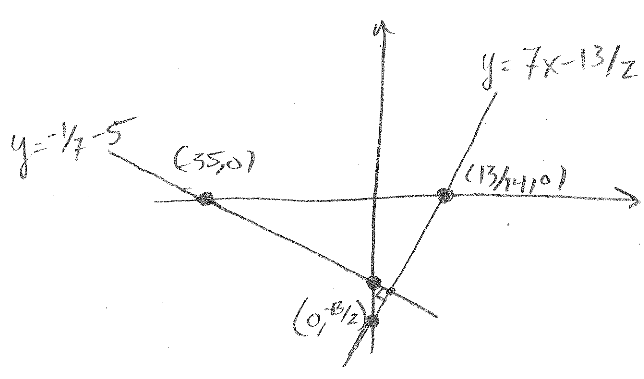
Hence the line is

$y-(-3) = 7(x - 1/2)$

$\Rightarrow y+3 = 7(x-1/2)$

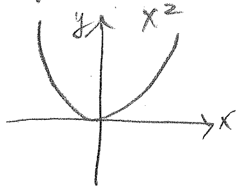
$\Rightarrow y = 7x - 7/2 - 3$

$\Rightarrow y = 7x - 7/2 - 6/2 = 7x - 13/2$.

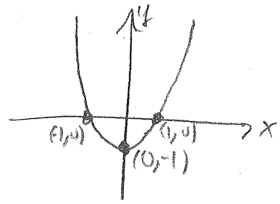


4.6: Graph the following functions

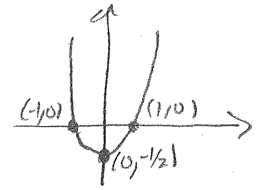
1) a) $y = x^2$



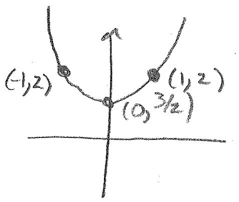
b) $y = x^2 - 1$



c) $y = \frac{1}{2}(x^2 - 1)$

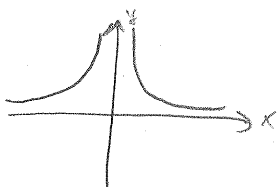


d) $y = \frac{1}{2}(x^2 - 1) + 2$

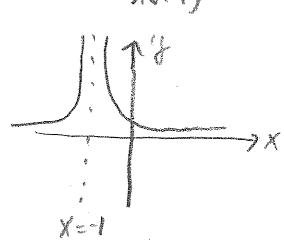


■

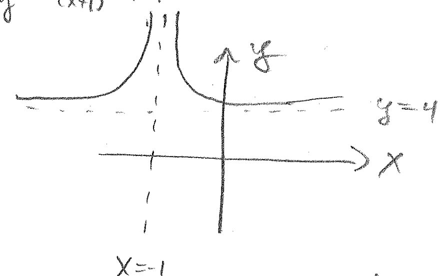
4) a) $y = \frac{1}{x^2}$



b) $y = \frac{1}{(x+1)^2}$



c) $y = \frac{1}{(x+1)^2} + 4$



■

4.7: Find the simultaneous solutions of the following system of equations.

1) a) $\begin{cases} 3x - 2y = 16 \\ 5x + y = 5 \end{cases}$

$3x - 2y = 16 \Rightarrow 2y = 3x - 16$
$\Rightarrow y = \frac{3}{2}x - 8$

$5x + y = 5 \Rightarrow y = -5x + 5$

$\frac{3}{2}x - 8 = -5x + 5$
$\Rightarrow 5x + 3/2x = 5 + 8 = 13$
$\Rightarrow \frac{10}{2}x + 3/2x = \frac{13}{2}x = 13$
$\Rightarrow x = \frac{2}{13}(13) = 2.$
$\Rightarrow y = -5(2) + 5 = -10 + 5 = -5.$

The point of intersection is $(2, -5)$.

b) $\begin{cases} x^2 - 4y = 6 \\ 2x + 2y = 3 \end{cases}$

$x^2 - 4y = 6 \Rightarrow 4y = x^2 - 6$
$\Rightarrow y = \frac{1}{4}x^2 - \frac{6}{4}$
$= \frac{1}{4}x^2 - \frac{3}{2}.$

$2x + 2y = 3 \Rightarrow 2y = -2x + 3$
$\Rightarrow y = -x + 3/2$

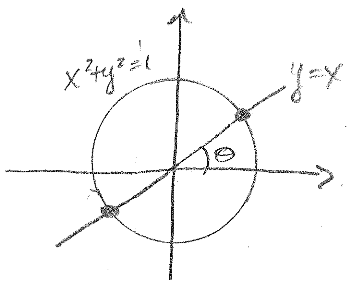
$\frac{1}{4}x^2 - \frac{3}{2} = -x + 3/2$
$\Rightarrow x^2 - 6 = -4x + 6$ (multiply both sides by 4)
$\Rightarrow x^2 + 4x - 12 = 0$
$\Rightarrow (x+6)(x-2) = 0$
$\Rightarrow x = -6$ or $x = 2.$

$2(-6) + 2y = 3 \Rightarrow 2y = 3 + 12 = 15 \Rightarrow y = 15/2.$
$2(2) + 2y = 3 \Rightarrow 2y = 3 - 4 = -1 \Rightarrow y = -1/2.$

Points of intersection: $(-6, 15/2)$ and $(2, -1/2)$. ■

3) Find the points of intersection for the line $y=x$ and the circle $x^2+y^2=1$. (Hint: Graph the functions).

The graph of the functions is



We observe that there should be two points of intersection, say (x_1,y_1) in the first quadrant and (x_2,y_2) in the third quadrant. Since they lie on the line $y=x$, we know $x_1=y_1$ and $x_2=y_2$. Moreover, the point (x_1,x_1) lies on the unit circle, so $\sin(\theta)=\cos(\theta)$ implies $\theta=\pi/4$ and so $x_1=\frac{1}{\sqrt{2}}$. By the geometry of the situation, we see $x_2=-1/\sqrt{2}$. Therefore the points of intersection are $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ and $(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}})$. ■

<u>Alternate solution</u>

Solve $x^2+y^2=1$ for y in terms of x in the first quadrant:

$$y=\sqrt{1-x^2}$$

then

$$x=\sqrt{1-x^2} \Rightarrow x^2=1-x^2$$
$$\Rightarrow 2x^2=1$$
$$\Rightarrow x^2=1/2$$
$$\Rightarrow x=\pm\sqrt{1/2}$$

so in the first quadrant we have the point $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$. In the third quadrant we have $y=-\sqrt{1-x^2}$, so

$$x=-\sqrt{1-x^2} \Rightarrow x^2=1-x^2 \Rightarrow 2x^2=1 \Rightarrow x^2=1/2 \Rightarrow x=-\sqrt{1/2}=-\frac{1}{\sqrt{2}}$$

because we have assumed x lies on the negative portion of the axis. Therefore the other point of intersection is $(-1/\sqrt{2},-1/\sqrt{2})$.

Rmk: This gives us a way of computing the values of $\sin(5\pi/4)$ and $\cos(5\pi/4)$. Namely, we observe in the picture above that the points $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and $(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$ lie on a line (the line $y=x$), and so their angles with the positive x-axis differ by π. Since we have $\sin(\pi/4) = \cos(\pi/4) = \frac{1}{\sqrt{2}}$, we obtain

$$\sin(\pi/4 + \pi) = \sin\left(\frac{5\pi}{4}\right) = \frac{-1}{\sqrt{2}}$$

and $$\cos(\pi/4 + \pi) = \cos(5\pi/4) = \frac{-1}{\sqrt{2}}$$