MATH 122
EXAM 03

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.** You may use a calculator **without a CAS** if you like, but a calculator is not necessary. By writing your name on the line below, you acknowledge that you have read and understand these directions.

Name: Solutions

Definitions	Points Earned	Points Possible	Problems	Points Earned	Points Possible
1		6	1		20
2		4	2		12
3		5	3		8
4		5	4		15
5		5	5		20
Subtotal		25	Subtotal		75
			Total		100

Date: April ~~12~~ 13, 2017.

1. Definitions

1 (6 Points). If $F'(t)$ is a continuous function on the interval $[a, b]$, then

$$\int_a^b F'(t)\,\mathrm{d}t = F(b) - F(a)$$

2 (4 Points). Assume that $\int f(x)\,\mathrm{d}x$ and $\int g(x)\,\mathrm{d}x$ exist.

(a)

$$\int f(x) \pm g(x)\,\mathrm{d}x = \int f(x)dx + \int g(x)dx$$

(b) Let a be a number.

$$\int af(x)\,\mathrm{d}x = a\int f(x)dx$$

3 (5 Points). Let $n \neq 1$ be a fixed number.

$$\int x^n\,\mathrm{d}x = \frac{1}{n+1}x^{n+1} + C$$

4 (5 Points).

$$\int e^x\,\mathrm{d}x = e^x + C$$

5 (5 Points).

$$\int \frac{1}{x}\,\mathrm{d}x = \ln|x| + C$$

2. Problems

1 (20 Points). Compute the following indefinite integrals:

(a) $\int 7\,dx = 7x + C$

(b) $\int 10x + 2\,dx = \int 10x\,dx + \int 2\,dx$

$= 10\int x\,dx + 2\int dx$

$= 10(\frac{1}{2}x^2) + 2x + C = 5x^2 + 2x + C$

(c) $\int 36x^2 + 26x\,dx = \int 36x^2\,dx + \int 26x\,dx$

$= 36\int x^2\,dx + 26\int x\,dx$

$= 36(\frac{1}{3}x^3) + 26(\frac{1}{2}x^2) + C$

$= 12x^3 + 13x^2 + C.$

(d) $\int x^2\,dx = \frac{1}{3}x^3 + C.$

(e) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{1/2}}{\frac{1}{2}} + C = 2\sqrt{x} + C.$

2 (12 Points). Compute the following indefinite integrals.

(a) $\int 25(x+7)^{24}\,\mathrm{d}x$ $\quad u = x+7,\ \frac{du}{dx} = 1 \Rightarrow du = dx$

$= \int 25u^{24}\,du = 25\int u^{24}\,du = 25\left(\frac{1}{25}u^{25}\right) + C = (x+7)^{25} + C.$

(b) $\int (x+2)e^{\frac{1}{2}x^2+2x+1}\,\mathrm{d}x$ $\quad u = \frac{1}{2}x^2 + 2x + 1,\ \frac{du}{dx} = x+2 \Rightarrow du = (x+2)dx$

$= \int e^{\frac{1}{2}x^2+2x+1}(x+2)dx = \int e^u\,du = e^u + C = e^{\frac{1}{2}x^2+2x+1} + C.$

(c) $\int \frac{4x}{2x^2+7}\,\mathrm{d}x$ $\quad u = 2x^2+7 \Rightarrow \frac{du}{dx} = 4x \Rightarrow du = 4x\,dx$

$\| $

$\int \frac{1}{2x^2+7}(4x)dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|2x^2+7| + C$

$= \ln(2x^2+7) + C.$

3 (8 Points). Compute the following indefinite integrals.

(a) $\int \frac{x}{\sqrt{x^2+1}}\,dx$ $\quad u = x^2+1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}du$

$$\| $$

$$\int \frac{1}{\sqrt{x^2+1}}\,x\,dx = \int \frac{1}{\sqrt{u}}\left(\frac{1}{2}\right)du = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\,\frac{u^{1/2}}{\frac{1}{2}} = \frac{1}{2}(2)\sqrt{u} + C$$

$$= \sqrt{x^2+1} + C.$$

(b) $\int 30e^{5x} - 2xe^{-x^2}\,dx$ $= \int 30e^{5x}dx + \int(-2x)e^{-x^2}dx$ $\quad u = -x^2, \Rightarrow du = -2x\,dx$

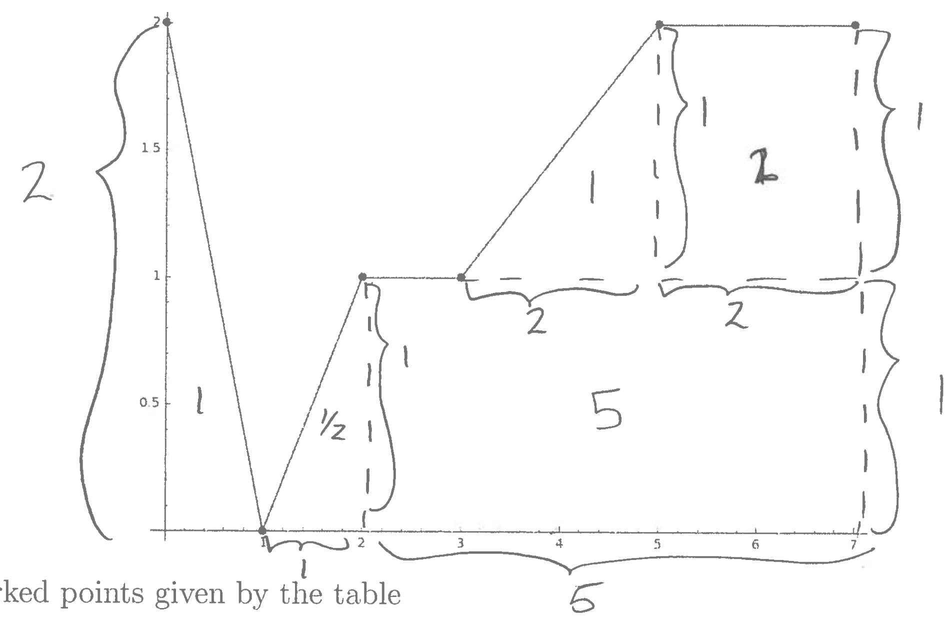
$$= 30\int e^{5x}dx + \int e^{-x^2}(-2x)dx$$

$$= 30\int e^{5x}dx + \int e^{u}\,du$$

$$= 30\left(\frac{1}{5}e^{5x}\right) + e^{u} + C$$

$$= 6e^{5x} + e^{-x^2} + C.$$

4 (15 Points). Consider the function f given by the graph



with marked points given by the table

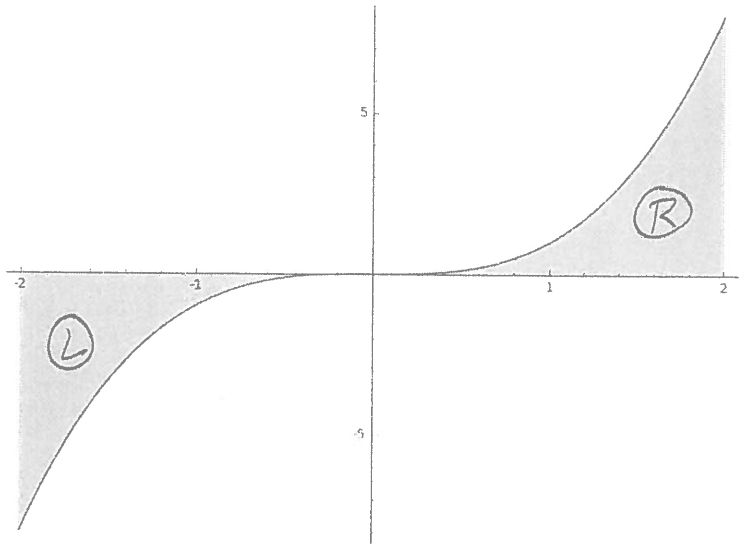
x	0	1	2	3	5	7
$f(x)$	2	0	1	1	2	2

Compute $\int_0^7 f(x)\,dx$.

To compute the integral, add the areas of the figures above to get

$$\int_0^7 f(x)dx = 1 + \tfrac{1}{2} + 1 + 5 + 2 = 9.5.$$

5 (20 Points). Find the **total** area between the graph of x^3 and the x-axis, between $x = -2$ and $x = 2$. That is, find the area of the shaded region below:



The total area is the sum of the two areas.

The right area is given by

$$\int_0^2 x^3 dx = \frac{1}{4}x^4\Big|_0^2 = \frac{1}{4}(2^4 - 0^4) = \frac{1}{4}(16) = 4$$

and the left area is given by

$$\int_{-2}^0 -x^3 dx = -\int_{-2}^0 x^3 dx = -\frac{1}{4}x^4\Big|_{-2}^0 = -\frac{1}{4}(0^4 - (-2)^4)$$
$$= -\frac{1}{4}(0-16)$$
$$= -\frac{1}{4}(-16) = 4.$$

Therefore the total area is $4+4=8$.