MATH 170
EXAM 01

BLAKE FARMAN
UNIVERSITY OF SOUTH CAROLINA

Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**

Name: Solutions - Version 1

Problem	Points Earned	Points Possible
1		10
2		20
3		12
4		18
5		20
6		20
Bonus		10
Total		100

Date: February 26, 2016.

1 (10 Points). *Let S be the set of all students at the University of South Carolina. Let A be the subset of all students not taking Math 170 this semester. Let B be the subset of all students majoring in business.*

(a) In words, what does the set $A \cup B$ represent?

The set of students either not taking Math 170 or majoring in business.

(b) In words, what does the set $A \cap B$ represent?

The set of students not taking Math 170 and majoring in business.

(c) In words, what do the sets $S \setminus A$ and $S \setminus B$ represent?

$S \setminus A$ is the set of students taking Math 170

$S \setminus B$ is the set of students not majoring in business.

(d) In words, what does the set $(S \setminus A) \cap (S \setminus B)$ represent?

The set of students taking Math 170 and not majoring in business.

(e) In words, what does the set $(S \setminus A) \cup (S \setminus B)$ represent?

The set of students taking Math 170 or not majoring in business.

2 (20 Points). *A bag contains two red marbles, two green marbles, two lavender marbles, one yellow marble, and one orange marble. The marbles are all distinguishable.*

(a) How many sets of four marbles include none of the red ones?

There are $2+2+2+1+1=8$ total marbles. There are 6 marbles that are not red, so there are

$$\binom{6}{4} = \frac{6!}{(6-4)!\,4!} = \frac{6!}{2!\,4!} = \frac{6\cdot 5\cdot 4!}{2\cdot 4!} = \frac{30}{2} = 15$$

ways to choose a set of four from these marbles.

(b) How many sets of four marbles include exactly one red marble?

There are

$$\binom{2}{1} = \frac{2!}{(2-1)!\,1!} = \frac{2!}{1!\,1!} = \frac{2}{1\cdot 1} = 2$$

ways to choose a red marble.

There are

$$\binom{6}{3} = \frac{6!}{(6-3)!\,3!} = \frac{6!}{3!\,3!} = \frac{6\cdot 5\cdot 4\cdot 3!}{3!\,3!} = \frac{6\cdot 5\cdot 4}{3!} = \frac{6\cdot 5\cdot 4}{6} = 20$$

ways to choose 3 marbles that are not red. Therefore, there are

$$2\cdot 20 = 40$$

ways to choose such a set.

3 (12 Points). *How many three letter sequences can be made using the six letters*

q, u, a, k, e?

$$P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1} = 5\cdot 4\cdot 3 = 5\cdot 12 = 60.$$

sequences.

Let $U = \{A, B, C, D, E, F, G\}$. Let $X = \{A, C, E, G\}$, $Y = \{B, C, D, E\}$, and $Z = \{C, D, F\}$. Use these sets to answer problems 4 and 5.

4 (18 Points). *Compute*

(a) $X \cap Y$,

$\{C, E\}$

(b) $X \cup Z$,

$\{A, C, D, E, F, G\}$

(c) The complement of Z *in* U, $U \setminus Z$.

$\{A, B, E, G\}$

5 (20 Points). *(a) What is the cardinality of $X \times Z$?*

$|X \times Z| = |X|\,|Z| = 4 \cdot 3 = 12$

(b) What is the cardinality of $Y \cup Z$?

$$|Y \cup Z| = |Y| + |Z| - |Y \cap Z|$$
$$= 4 + 3 - 2$$
$$= 5$$

6 (20 Points). *Use a truth table to prove the following logical equivalences.*

(a)

$$\neg p \vee q \equiv p \implies q.$$

p	q	$\neg p$	$\neg p \vee q$	$p \Rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

These columns are the same, so

$\neg p \vee q \equiv p \Rightarrow q$.

(b)

$$p \vee (p \wedge q) \equiv p.$$

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

These columns are the same, so

$p \equiv p \vee (p \wedge q)$.