MATH 170
EXAM 01

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**

Name: Solutions - Version 2

Problem	Points Earned	Points Possible
1		10
2		20
3		12
4		18
5		20
6		20
Bonus		10
Total		100

Date: February 26, 2016.

1 (10 Points). *Let S be the set of all students at the University of South Carolina. Let A be the subset of all students taking Math 170 this semester. Let B be the subset of all students not majoring in business.*

(a) In words, what does the set $A \cup B$ represent?

The set of students taking 170 or not majoring in Business.

(b) In words, what does the set $A \cap B$ represent?

The set of students taking 170 and not majoring in business.

(c) In words, what do the sets $S \setminus A$ and $S \setminus B$ represent?

$S \setminus A$ is the set of students not taking 170

$S \setminus B$ is the set of students majoring in business.

(d) In words, what does the set $(S \setminus A) \cap (S \setminus B)$ represent?

The set of students not taking 170 and majoring in business.

(e) In words, what does the set $(S \setminus A) \cup (S \setminus B)$ represent?

The set of students not taking 170 or are majoring in business.

2 (20 Points). *A bag contains three red marbles, two green marbles, one lavender marbles, one yellow marble, and one orange marble. The marbles are all distinguishable.*

(a) How many sets of four marbles include none of the red ones?

There are

$$3+2+1+1+1=8$$

total marbles. There are 5 marbles once the reds have been removed, so there are

$$\binom{5}{4}=\frac{5!}{(5-4)!\,4!}=\frac{5!}{1!\,4!}=\frac{5\cdot 4!}{4!}=5$$

ways to choose such a set.

(b) How many sets of four marbles include exactly one red marble?

There are

$$\binom{3}{1}=\frac{3!}{(3-1)!\,1!}=\frac{3!}{2!\,1!}=\frac{3\cdot 2\cdot 1}{2\cdot 1\cdot 1}=3$$

ways to choose one red marble and

$$\binom{5}{3}=\frac{5!}{(5-3)!\,3!}=\frac{5!}{2!\,3!}=\frac{5\cdot 4\cdot 3!}{2\cdot 3!}=\frac{20}{2}=10$$

ways to choose the other three. Therefore there are

$$3\cdot 10=30$$

ways to choose such a set

3 (12 Points). *How many two letter sequences can be made using the six letters*

q, u, a, k, e?

$$P(5,2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\cdot 4\cdot 3!}{3!} = 5\cdot 4 = 20.$$

Let $U = \{A, B, C, D, E, F, G\}$. Let $X = \{B, D, F\}$, $Y = \{A, F, G\}$, and $Z = \{A, B, E, G\}$. Use these sets to answer problems 4 and 5.

4 (18 Points). *Compute*

(a) $X \cap Y$,

$\{F\}$

(b) $X \cup Z$,

$\{A, B, D, E, F, G\}$

(c) The complement of Z *in* U, $U \setminus Z$.

$\{C, D, F\}$

5 (20 Points). *(a) What is the cardinality of $X \times Z$?*

$$|X \times Z| = |X||Z| = 3 \cdot 4 = 12$$

(b) What is the cardinality of $Y \cup Z$?

$$\begin{aligned} |Y \cup Z| &= |Y| + |Z| - |Y \cap Z| \\ &= 3 + 4 - 2 \\ &= 5. \end{aligned}$$

6 (20 Points). *Use a truth table to prove the following logical equivalences.*

(a)

$$\neg p \vee q \equiv \neg q \implies \neg p.$$

p	q	$\neg p$	$\neg p \vee q$	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
T	T	F	T	F	F	T
T	F	F	F	T	F	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

These columns are the same, so

$$\neg p \vee q \equiv \neg q \Rightarrow \neg p.$$

(b)

$$p \wedge (p \vee q) \equiv p.$$

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

These columns are the same, so

$$p \wedge (p \vee q) \equiv p.$$