ALGEBRA REVIEW

BLAKE FARMAN

Lafayette College

Name: Solutions

1. Find all **real** solutions to each equation.

(a) $x^2 - 8x + 12 = 0$

$$0 = x^2 - 8x + 12 = (x-2)(x-6)$$

$$\iff x - 2 = 0 \quad \text{or} \quad x - 6 = 0$$

$$\iff x = 2 \quad \text{or} \quad x = 6$$

(b) $2x^2 - 9x = 5$

$$2x^2 - 9x = 5 \iff 0 = 2x^2 - 9x - 5 = (2x+1)(x-5)$$

$$\iff 2x + 1 = 0 \quad \text{or} \quad x - 5 = 0$$

$$\iff 2x = -1 \quad \text{or} \quad x = 5$$

$$\iff x = \frac{-1}{2} \quad \text{or} \quad x = 5$$

(c) $x^2 - 1 = 0$

$0 = x^2 - 1 = (x+1)(x-1) \iff x+1 = 0$ or $x - 1 = 0$

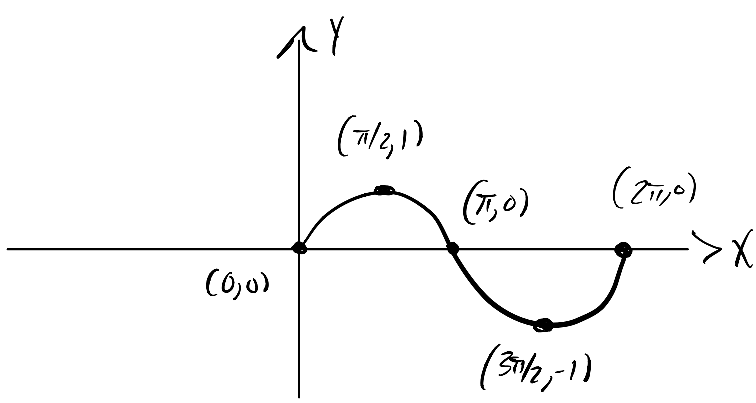
$\iff x = -1$ or $x = 1$

(d) $x^2 = 2$

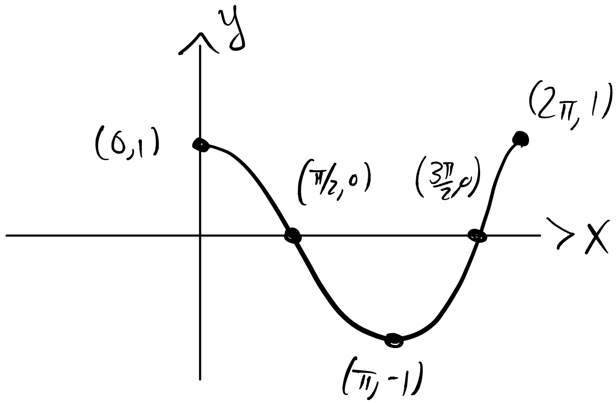
$x = \pm\sqrt{2}$

2. Sketch a graph of the following functions:

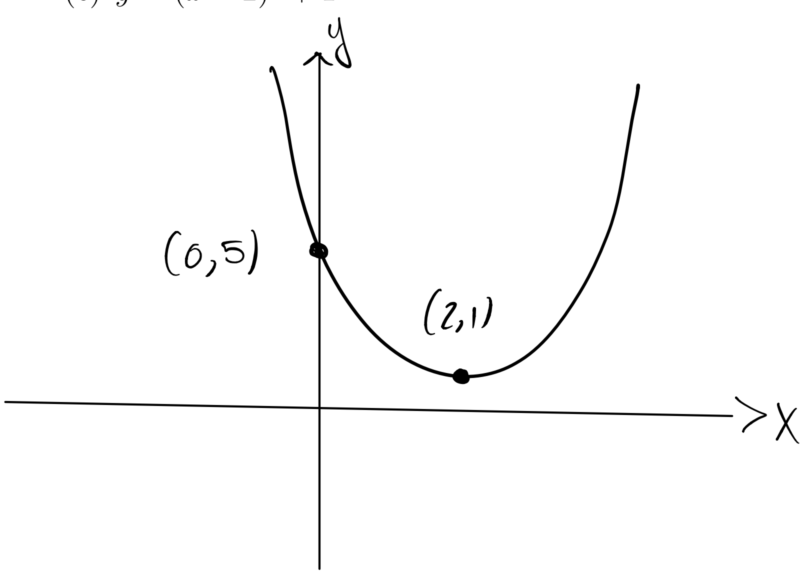
(a) $y = \sin(x)$



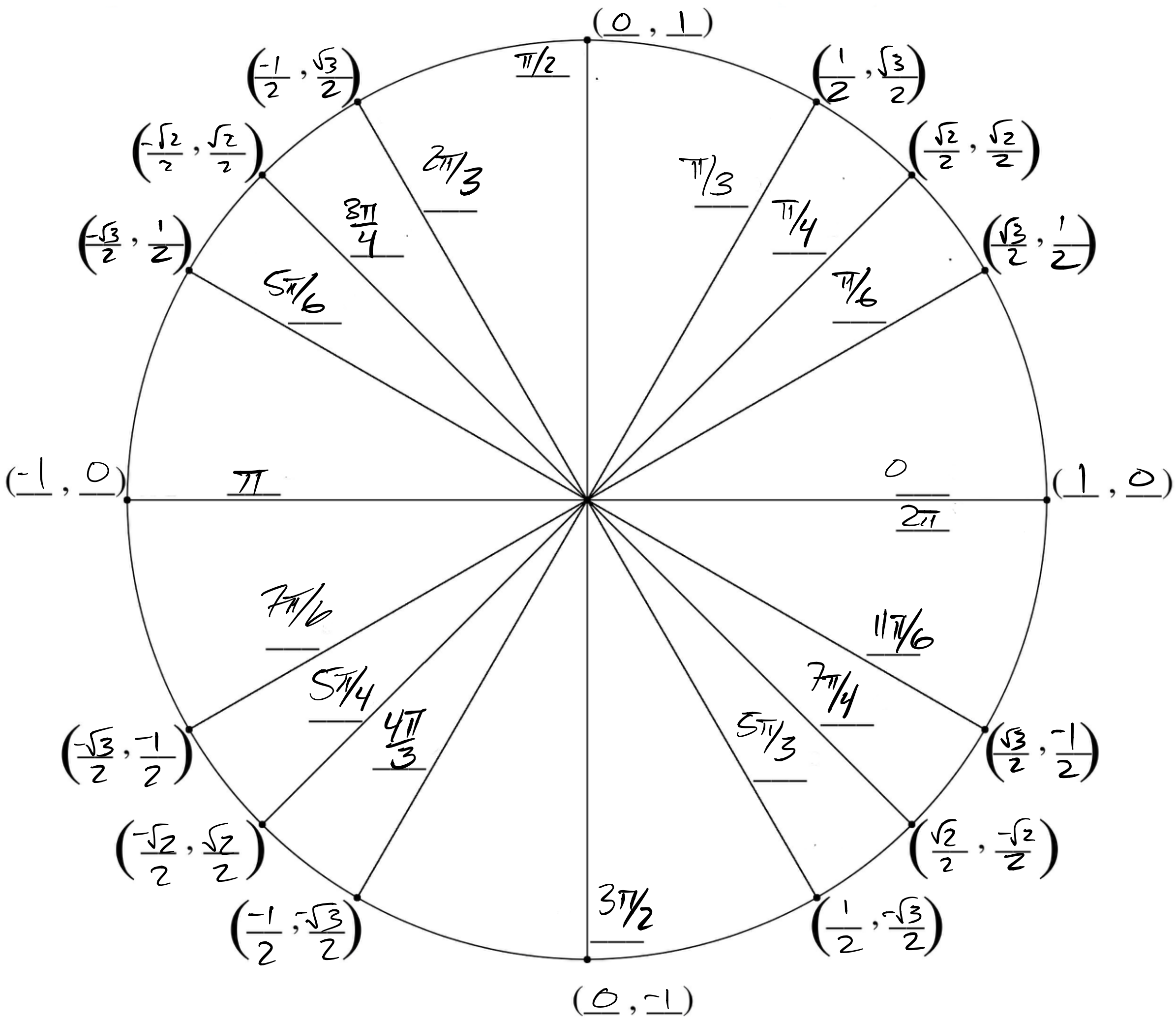
(b) $y = \cos(x)$



(c) $y = (x-2)^2 + 1$



3. Fill in the unit circle below with angle measurements in **radians** and the corresponding values of cosine and sine.



4. Simplify the following expressions:

(a) $\dfrac{\frac{xy}{x+y}}{\frac{x^2y}{(x+y)^3}}$ $= \dfrac{xy}{x+y} \cdot \dfrac{(x+y)^3}{x^2y} = \dfrac{x}{x^2} \cdot \dfrac{y}{y} \cdot \dfrac{(x+y)^3}{(x+y)}$

$= \dfrac{1}{x} \cdot 1 \cdot (x+y)^2$

$= \boxed{\dfrac{(x+y)^2}{x}}$

(b) $\dfrac{\frac{xy}{x-y}}{\frac{x^2}{y} \cdot \frac{y^3}{x}}$ $= \dfrac{\frac{xy}{(x-y)}}{\frac{x^2y^3}{yx}} = \dfrac{xy}{(x-y)} \cdot \dfrac{yx}{x^2y^3}$

$= \dfrac{x^2y^2}{(x-y)x^2y^3} = \dfrac{x^2}{x^2} \cdot \dfrac{y^2}{y^3} \cdot \dfrac{1}{x-y}$

$= 1 \cdot \dfrac{1}{y} \cdot \dfrac{1}{x-y} = \boxed{\dfrac{1}{y(x-y)}}$

(c) $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}} = \dfrac{\frac{y}{y}\left(\frac{1}{x}\right) - \frac{x}{x}\left(\frac{1}{y}\right)}{\frac{y}{y}\left(\frac{1}{x}\right) + \frac{x}{x}\left(\frac{1}{y}\right)} = \dfrac{\frac{y}{xy} - \frac{x}{xy}}{\frac{y}{xy} + \frac{x}{xy}}$

$$= \frac{\frac{y-x}{xy}}{\frac{x+y}{xy}} = \frac{y-x}{xy}\left(\frac{xy}{x+y}\right) = \boxed{\frac{y-x}{x+y}}$$

(d) $\dfrac{4yz}{x^2} - \dfrac{2z}{xy^2} + \dfrac{1}{xyz} = \dfrac{y^2z}{y^2z}\left(\dfrac{4yz}{x^2}\right) - \dfrac{xz}{xz}\left(\dfrac{2z}{xy^2}\right) + \dfrac{xy}{xy}\left(\dfrac{1}{xyz}\right)$

$$= \frac{4y^3z^2}{x^2y^2z} - \frac{2xz^2}{x^2y^2z} + \frac{xy}{x^2y^2z}$$

$$= \boxed{\frac{4y^3z^2 - 2xz^2 + xy}{x^2y^2z}}$$

5. $2x(y-3) - y(x+xy) + 2y(x+1)$

$$= 2xy - 6x - xy - xy^2 + 2xy + 2y$$
$$= 2xy - xy + 2xy - 6x - xy^2 + 2y$$
$$= (2-1+2)xy - 6x - xy^2 + 2y$$
$$= \boxed{3xy - 6x - xy^2 + 2y}$$

6. $x(y+z) - z(x+y) + 2y(x-z) - x(3y-2z)$

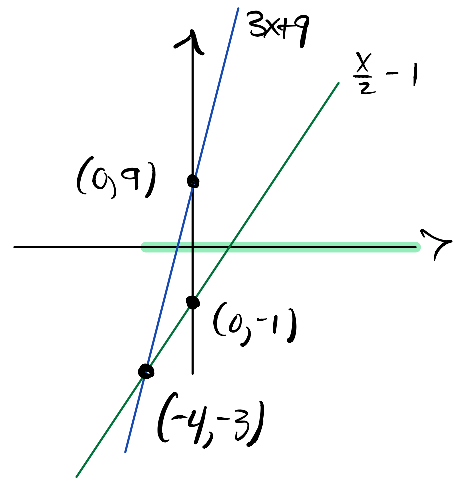
$$= xy + xz - xz - yz + 2xy - 2yz - 3xy + 2xz$$
$$= xy + 2xy - 3xy + xz - xz + 2xz - yz - 2yz$$
$$= (1+2-3)xy + (1-1+2)xz + (-1-2)yz$$
$$= (0)xy + 2xz + (-3)yz$$
$$= \boxed{2xz - 3yz}$$

7. Solve the following inequalities:

(a) $\frac{x}{2} - 1 < 3x + 9$

$$\frac{x}{2} - 1 < 3x + 9 \iff x - 2 < 6x + 18$$
$$\iff -18 - 2 < 6x - x$$
$$\iff -20 < 5x$$
$$\iff -\frac{20}{5} = \boxed{-4 < x}$$

Geometric:



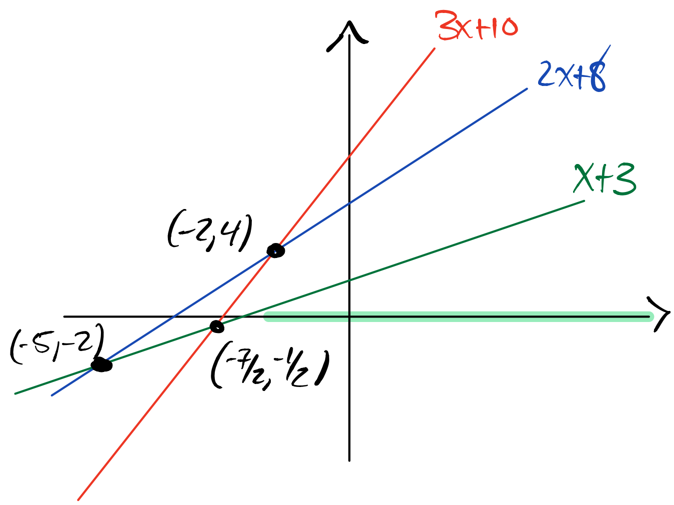
(b) $x + 3 < 2x + 8 < 3x + 10 \iff 3 < x + 8 < 2x + 10$

$3 < x + 8 \iff 3 - 8 = -5 < x$ and

$x + 8 < 2x + 10 \iff 8 < x + 10$
$\iff 8 - 10 = -2 < x$

$-5 < \boxed{-2 < x}$

Geometric:



(c) $|2x - 5| \leq 11 \iff -11 \leq 2x - 5 \leq 11$
$\iff -6 \leq 2x \leq 16$
$\iff -3 \leq x \leq 8$

Geometric: