CHAIN RULE

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Name: _____

Theorem (The Chain Rule)**.** *Assume that g is differentiable at x and f is differentiable at $g(x)$. Then composition of f with g, $f \circ g(x) = f\big(g(x)\big)$, is differentiable at x and*

$$\frac{\mathrm{d}}{\mathrm{d}x} f \circ g(x) = f'\big(g(x)\big) \cdot g'(x).$$

1. Let $f(x) = (3x^2 + 1)^2$.

(a) Expand $f(x)$, then take the derivative.

(b) Write $f(x) = (3x^2 + 1)^2 = (3x^2 + 1)(3x^2 + 1)$ and apply the Product Rule.

(c) Apply the chain rule directly to $f(x)$.

(d) Are your answers in parts (a), (b), and (c) the same? Why or why not?

2. Assume that f is a differentiable function and let $g(x) = \left(f\left(\sqrt{x}\right)\right)^3$.

(a) Compute $g'(x)$. Your answer should include both f and f'.

(b) If $f(2) = 1$ and $f'(2) = -2$, calculate $g'(4)$.

Find the derivative of the given function.

3. $f(x) = (1 + x + x^2)^{99}$

4. $g(\theta) = (2 - \sin(\theta))^{3/2}$

5. $g(\theta) = \cos^2(\theta)$

6. $f(\theta) = \cot^2(\sin(\theta))$