CURVE SKETCHING

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Name: Solutions

1. Sketch the curve

$$f(x) = 3x^4 - 8x^3 + 6x^2$$

(a) State the domain of f.

$(-\infty, \infty)$

(b) Find the intercepts and express them as an (x, y) pair. Write NONE if there are none.

x-intercept(s): $(0,0)$

y-intercept: $(0,0)$

$3x^4 - 8x^3 + 6x^2 = x^2(3x^2 - 8x + 6)$

$D = (-8)^2 - 4(3)(6) = 64 - 72 < 0$, so $3x^2 - 8x + 6$ irreducible.

(c) Is the function even, odd, or neither? What type of symmetry does the function have?

$f(-x) = 3(-x)^4 - 8(-x)^3 + 6(-x)^2$

$= 3x^4 + 8x^3 + 6x^2$

$\neq f(x), -f(x)$

So neither even nor odd, no symmetry.

(d) Find the asymptotes. Write NONE if there are none.

Horizontal: NONE

Vertical: NONE

(e) Find the intervals where the function is increasing and decreasing. Write NONE if not applicable.

Increasing: $(0,1) \cup (1,\infty)$

Decreasing: $(-\infty, 0)$

(f) State the local maximum and local minimum value(s). Write NONE if not applicable.

Local maximum value(s): NONE

Local minimum value(s): $(0,0)$

(g) Find the intervals on which the function is concave up and concave down. State the inflection points. Write NONE if not applicable.

Concave Up: $(-\infty, 1/3) \cup (1,\infty)$

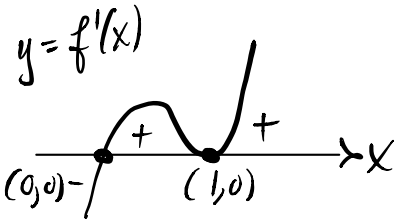
Concave Down: $(1/3, 1)$

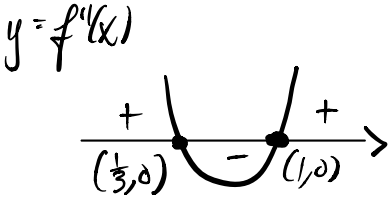
Inflection Points: $(1/3, \frac{33}{81})$, $(1,1)$

$f(x) = 3x^4 - 8x^3 + 6x^2$

$f'(x) = 12x^3 - 24x^2 + 12x = 12x(x^2 - 2x + 1) = 12x(x-1)^2$

$f''(x) = 36x^2 - 48x + 12 = 12(3x^2 - 4x + 1) = 12(3x-1)(x-1)$





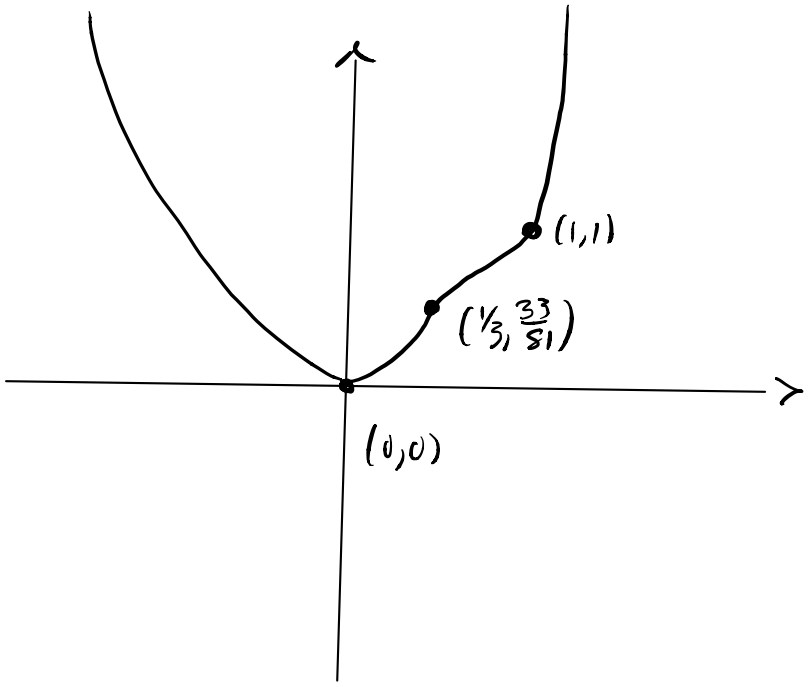
$f(1/3) = \frac{3}{81} - \frac{8}{27} + \frac{6}{9}$

$= \frac{3 - 24 + 54}{81} = \frac{33}{81}$

$f(1) = 3 - 8 + 6$

$= 1$

(h) Use your answers to Parts (a)-(g) to sketch the curve. Be sure that your graph is labeled and neat.



2. Sketch the curve

$$f(x) = \frac{2x^2}{x^2 - 1}$$

(a) State the domain of f.

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(b) Find the intercepts and express them as an (x, y) pair. Write NONE if there are none.

x-intercept(s): $(0, 0)$

y-intercept: $(0, 0)$

(c) Is the function even, odd, or neither? What type of symmetry does the function have?

Even; Symmetry about y-axis:

$$f(-x) = \frac{2(-x)^2}{(-x)^2 - 1} = \frac{2x^2}{x^2 - 1} = f(x)$$

(d) Find the asymptotes. Write NONE if there are none.

Horizontal: $y = 2$

Vertical: $x = -1,\ x = 1$

(e) Find the intervals where the function is increasing and decreasing. Write NONE if not applicable.

Increasing: $(-\infty, -1) \cup (-1, 0)$

Decreasing: $(0, 1) \cup (1, \infty)$

(f) State the local maximum and local minimum value(s). Write NONE if not applicable.

Local maximum value(s): $(0, 0)$

Local minimum value(s): NONE

(g) Find the intervals on which the function is concave up and concave down. State the inflection points. Write NONE if not applicable.

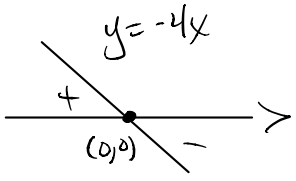
Concave Up: $(-\infty, -1) \cup (1, \infty)$

Concave Down: $(-1, 1)$

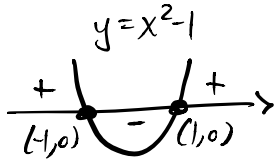
Inflection Points: NONE

$$\lim_{x\to\infty} \frac{2x^2}{x^2-1} = \lim_{x\to\infty} \frac{2}{1-1/x^2} = \frac{2}{1-0} = 2$$

$$f'(x) = \frac{4x(x^2-1) - 2x^2(2x)}{(x^2-1)^2} = \frac{4x^3 - 4x - 4x^3}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$



$$f''(x) = \frac{-4(x^2-1)^2 - (-4x)(2)(x^2-1)(2x)}{(x^2-1)^4} = \frac{-4x^2+4+16x^2}{(x^2-1)^3} = \frac{12x^2+4}{(x^2-1)^3}$$



(h) Use your answers to Parts (a)-(g) to sketch the curve. Be sure that your graph is labeled and neat.