IMPLICIT DIFFERENTIATION

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Name: Solutions

Use implicit differentiation to find $\mathrm{d}y/\,\mathrm{d}x$.

1. $2x^2 + xy - y^2 = 2$

$4x + (y + xy') - 2yy' = 0$

$\Rightarrow 4x + y = 2yy' - xy' = (2y - x)y'$

$\Rightarrow y' = \boxed{\frac{4x+y}{2y-x}}$

2. $x^3 - xy^2 + y^3 = 1$

$3x^2 - (y^2 + 2xyy') + 3y^2y' = 0$

$\Rightarrow 3x^2 - y^2 - 2xyy' + 3y^2y' = 0$

$\Rightarrow 3x^2 - y^2 = 2xyy' - 3y^2y' = (2xy - 3y^2)y'$

$\Rightarrow y' = \boxed{\frac{3x^2 - y^2}{2xy - 3y^2}}$

3. $\cos(xy) = 1 + \sin(y)$

$$-\sin(xy)(y + xy') = y'\cos(y)$$

$$\Rightarrow -y\sin(xy) - xy'\sin(xy) = y'\cos(y)$$

$$\Rightarrow -y\sin(xy) = xy'\sin(xy) + y'\cos(y) = y'(x\sin(xy) + \cos(y))$$

$$\Rightarrow y' = \boxed{\frac{-y\sin(xy)}{x\sin(xy) + \cos(y)}}$$

4. $xy = \sqrt{x^2 + y^2}$

$$y + xy' = \frac{1}{2}(x^2+y^2)^{-1/2}(2x + 2yy') = \frac{2(x+yy')}{2\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}} + \frac{yy'}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \frac{xy' - yy'}{\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}} - y \Rightarrow y'(x-y) = x - y\sqrt{x^2+y^2}$$

$$\Rightarrow \boxed{y' = \frac{x - y\sqrt{x^2+y^2}}{(x-y)}}$$

5. You are given that $f(1) = 2$ and $f(x) + x^2 f(x)^3 = 10$. Find $f'(1)$.

$$f'(x) + 2xf(x)^3 + 3x^2 f(x)^2 f'(x) = 0$$

$$\Rightarrow f'(x) + 3x^2 f(x)^2 f'(x) = -2xf(x)^3$$

$$\Rightarrow f'(x)(1 + 3x^2 f(x)^2) = -2xf(x)^3$$

$$\Rightarrow f'(x) = \frac{-2xf(x)^3}{1 + 3x^2 f(x)^2}$$

Setting $x = 1$ we get

$$f'(1) = \frac{-2(1)f(1)^3}{1 + 3(1)^2 f(1)^2} = \frac{-2(2)^3}{1 + 3(2)^2} = \boxed{\frac{-16}{13}}$$