**INVERSE FUNCTIONS**

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Name: Solutions

**Laws of Exponents.** Let $a, b \neq 1$ be positive numbers. If $x$ and $y$ are any real numbers, then

$b^{x+y} = b^x b^y$ $\qquad$ $b^{x-y} = \frac{b^x}{b^y}$ $\qquad$ $(b^x)^y = b^{xy}$ $\qquad$ $(ab)^x = a^x b^x$

Simplify the following expressions.

**1.** $\dfrac{4^{-3}}{2^{-2}}$

$$\frac{4^{-3}}{2^{-2}} = \frac{2^2}{4^3} = \frac{4}{4^3} = \frac{1}{4^2} = \boxed{\frac{1}{16}}$$

**2.** $8^{4/3}$

$$8^{4/3} = \left(\sqrt[3]{8}\right)^4 = 2^4 = \boxed{16}$$

**3.** $x(3x^2)^3$

$$x(3x^2)^3 = x(27)x^6 = \boxed{27x^7}$$

**4.** $b^8(2b^4)$

$$b^8(2b^4) = 2b^{8+4} = \boxed{2b^{12}}$$

**Laws of Logarithms.** Let $a, b \neq 1$ be positive numbers. If $x$ and $y$ are positive numbers, then

$$\log_b(xy) = \log_b(x) + \log_b(y) \qquad \log_b(x^r) = r\log_b(x)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y) \qquad \log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Find the given logarithm.

**5.** $\log_9(1)$

$\boxed{0}$

**8.** $\log_7(1)$

$\boxed{0}$

**11.** $\log_3\left(\frac{1}{27}\right)$

$\log_3\left(\frac{1}{27}\right) = \log_3(1) - \log_3(27)$

$= 0 - 3$

$= \boxed{-3}$

**6.** $\log_9(9^8)$

$\boxed{8}$

**9.** $\log_7(49)$

$\log_7(49) = \log_7(7^2)$

$= \boxed{2}$

**12.** $\log_{10}(\sqrt{10})$

$\log_{10}(\sqrt{10}) = \log_{10}(10^{1/2})$

$= \boxed{1/2}$

**7.** $\log_9(9)$

$\boxed{1}$

**10.** $\log_7\left(\frac{1}{49}\right)$

$\log_7\left(\frac{1}{49}\right) = \log_7(1) - \log_7(7^2)$

$= 0 - 2$

$= \boxed{-2}$

**13.** $\log_5(0.2)$

$\log_5(0.2) = \log_5\left(\frac{2}{10}\right)$

$= \log_5\left(\frac{1}{5}\right)$

$= \log_5(1) - \log_5(5)$

$= 0 - 1$

$= \boxed{-1}$

Expand the given expression.

**14.** $\log_5\left(\frac{x}{2}\right) = \boxed{\log_5(x) - \log_5(2)}$

**15.** $\log_3\left(x\sqrt{y}\right) = \log_3(x) + \log_3(\sqrt{y})$

$= \boxed{\log_3(x) + \frac{1}{2}\log_3(y)}$

**16.** $\log_3(5a) = \boxed{\log_3(5) + \log_3(a)}$

**17.** $\log_5\left(\frac{2a}{b}\right) = \log_5(2a) - \log_5(b)$

$= \boxed{\log_5(2) + \log_5(a) - \log_5(b)}$

**18.** $\log_{10}\left((w^2z)^{10}\right) = 10\log_{10}(w^2z)$

$= 10\left(2\log_{10}(w) + \log_{10}(z)\right)$

$= \boxed{20\log_{10}(w) + 10\log_{10}(z)}$

**19.** $\log_7\left(\frac{\sqrt[3]{wz}}{x}\right) = \log_7\left(\sqrt[3]{wz}\right) - \log_7(x)$

$= \frac{1}{3}\log_7(wz) - \log_7(x)$

$= \boxed{\frac{1}{3}\log_7(w) + \frac{1}{3}\log_7(z) - \log_7(x)}$

Combine the given expression.

**20.** $4\log_2(x) - \frac{1}{3}\log_2(x^2+1) = \log_2(x^4) - \log_2\left(\sqrt[3]{x^2+1}\right)$

$= \boxed{\log_2\left(\frac{x^4}{\sqrt[3]{x^2+1}}\right)}$

**21.** $\log_{10}(5) + 2\log_{10}(x) + 3\log_{10}(x^2+5) = \boxed{\log_{10}\left(5x^2(x^2+5)^3\right)}$

**22.** $2\log_8(x+1) + 2\log_8(x-1) = \log_8\left((x+1)^2(x-1)^2\right)$

$= \log_8\left((x^2-1)^2\right)$

**23.** $\log_5(x^2-1) - \log_5(x-1) = \log_5\left(\frac{x^2-1}{x-1}\right) = \boxed{\log_5(x+1)}$