LIMITS

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Name: Solutions

In each of the problems, evaluate the limit if it exists. Indicate any limit laws that you use. If the limit does not exist, explain why.

1. Use the limits

$$\lim_{x\to 2} f(x) = 4 \qquad \lim_{x\to 2} g(x) = -2 \qquad \lim_{x\to 2} h(x) = 0$$

to complete each of the following.

(a) $\lim_{x\to 2}[f(x) + 5g(x)]$

$= \lim_{x\to 2} f(x) + \lim_{x\to 2} 5g(x)$

$= \lim_{x\to 2} f(x) + 5\lim_{x\to 2} g(x)$

$= 4+5(-2) = 4-10$

$= \boxed{-6}$

(c) $\lim_{x\to 2}\sqrt{f(x)}$

$= \sqrt{\lim_{x\to 2} f(x)}$

$= \sqrt{4}$

$= \boxed{2}$

(e) $\lim_{x\to 2}\dfrac{g(x)}{h(x)}$

$\boxed{\text{Does Not Exist}}$

(b) $\lim_{x\to 2}[g(x)]^3$

$= \left[\lim_{x\to 2} g(x)\right]^3$

$= [-2]^3$

$= \boxed{-8}$

(d) $\lim_{x\to 2}\dfrac{3f(x)}{g(x)}$

$= \dfrac{\lim_{x\to 2} 3f(x)}{\lim_{x\to 2} g(x)}$

$= \dfrac{3\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)}$

$= \dfrac{3(4)}{-2} = \boxed{-6}$

(f) $\lim_{x\to 2}\dfrac{g(x)h(x)}{f(x)}$

$= \dfrac{\lim_{x\to 2} g(x)h(x)}{\lim_{x\to 2} f(x)}$

$= \dfrac{\lim_{x\to 2} g(x)\lim_{x\to 2} h(x)}{\lim_{x\to 2} f(x)}$

$= \dfrac{(-2)(0)}{4} = \boxed{0}$

2. $\lim_{x\to -1}(x^4-3x)(x^2+5x+3)$

$= ((-1)^4-3(-1))((-1)^2+5(-1)+3)$

$= (1+3)(1-5+3)$

$= 4(-1)$

$= \boxed{-4}$

3. $\lim_{u\to -2}\sqrt{u^4+3u+6}$

$= \sqrt{(-2)^4+3(-2)+6}$

$= \sqrt{16-6+6}$

$= \sqrt{16}$

$= \boxed{4}$

4. $\lim_{t\to 2}\left(\frac{t^2-2}{t^3-3t+5}\right)^2$

$= \left(\frac{2^2-2}{2^3-3(2)+5}\right)^2$

$= \left(\frac{4-2}{8-6+5}\right)^2$

$= \left(\frac{2}{7}\right)^2$

$= \boxed{4/49}$

5. $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$ $= \lim_{t\to -3} \dfrac{(t+3)(t-3)}{(2t+1)(t+3)}$

$= \lim_{t\to -3} \dfrac{t-3}{2t+1}$

$= \dfrac{-3-3}{2(-3)+1}$

$= \dfrac{-6}{-6+1} = \boxed{6/5}$

6. $\lim_{x\to -3} \dfrac{x^2+3x}{x^2-x-12}$ $= \lim_{x\to -3} \dfrac{x(x+3)}{(x+3)(x-4)}$

$= \lim_{x\to -3} \dfrac{x}{x-4}$

$= \dfrac{-3}{-3-4}$

$= \dfrac{-3}{-7} = \boxed{\dfrac{3}{7}}$

7. $\lim_{x\to -2} \dfrac{x+2}{x^3+8}$

[Hint: $x^3+a^3 = (x+a)(x^2-ax+a^2)$]

$\lim_{x\to -2} \dfrac{x+2}{x^3+8} = \lim_{x\to -2} \dfrac{x+2}{(x+2)(x^2-2x+4)}$

$= \lim_{x\to -2} \dfrac{1}{x^2-2x+4}$

$= \dfrac{1}{(-2)^2-2(-2)+4}$

$= \dfrac{1}{4+4+4} = \boxed{\dfrac{1}{12}}$

8. $\lim\limits_{h\to 0} \dfrac{(2+h)^3 - 8}{h}$ $= \lim\limits_{h\to 0} \dfrac{2^3 + 3(2)^2 h + 3(2)h^2 + h^3 - 8}{h}$

$$= \lim_{h\to 0} \frac{8 + 12h + 6h^2 + h^3 - 8}{h}$$

$$= \lim_{h\to 0} \frac{12h + 6h^2 + h^3}{h}$$

$$= \lim_{h\to 0} 12 + 6h + h^2 = 12 + 0 + 0 = \boxed{12}$$

9. $\lim\limits_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right)$ $= \lim\limits_{t\to 0} \left[\dfrac{t+1}{t(t+1)} - \dfrac{1}{t(t+1)}\right]$

$$= \lim_{t\to 0} \frac{t+1-1}{t(t+1)}$$

$$= \lim_{t\to 0} \frac{t}{t(t+1)} = \lim_{t\to 0} \frac{1}{t+1} = \frac{1}{0+1} = \boxed{1}$$

10. $\lim\limits_{t\to 0} \left(\dfrac{\sqrt{1+t} - \sqrt{1-t}}{t}\right)$ $= \lim\limits_{t\to 0} \dfrac{\sqrt{1+t} - \sqrt{1-t}}{t} \cdot \dfrac{\sqrt{1+t} + \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}}$

$$= \lim_{t\to 0} \frac{(1+t) - (1-t)}{t(\sqrt{1+t} + \sqrt{1-t})}$$

$$= \lim_{t\to 0} \frac{1+t-1+t}{t(\sqrt{1+t} + \sqrt{1-t})}$$

$$= \lim_{t\to 0} \frac{2t}{t(\sqrt{1+t} + \sqrt{1-t})}$$

$$= \lim_{t\to 0} \frac{2}{\sqrt{1+t} + \sqrt{1-t}}$$

$$= \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = \frac{2}{1+1} = \frac{2}{2} = \boxed{1}$$