MEAN VALUE THEOREM

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Theorem (Mean Value). Let f be a function that satisfies the following hypotheses:

- (1) f is continuous on the closed interval [a, b].
- (2) f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a).$$

In Problems 1 through 4, verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval and find all numbers c that satisfy its conclusion.

1.
$$f(x) = x^3 - x^2 - 6x + 2$$
, [0,3]

2. $f(x) = \cos(2x), \ [\pi/8, 7\pi/8]$

3.
$$f(x) = 3x^2 + 2x + 5$$
, $[-1, 1]$

4.
$$f(x) = \frac{x}{x+2}$$
, [1,4]