PRODUCT AND QUOTIENT RULES

BLAKE FARMAN

Lafayette College

Name: Solutions

In each of the problems, use the

Product Rule. If f and g are differentiable functions, then

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$$

and

Quotient Rule. If f and g are differentiable functions, then

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

to compute the derivative. Use proper notation and simplify your final answers. In some cases it might be advantageous to simplify/rewrite first. **Do not use rules found in later sections.**

1. Let $f(x) = g(x)h(x)$, $g(10) = -4$, $h(10) = 560$, $g'(10) = 0$, $h'(10) = 4$. Find $f'(10)$.

$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

$$\Rightarrow f'(10) = g'(10)h(10) + g(10)h'(10)$$

$$= 0(560) + (-4)(4)$$

$$= \boxed{-16}$$

2. Let $z(-3) = 6$, $z'(-3) = 15$, and $y(x) = \dfrac{z(x)}{1+x^2}$. Find $y'(-3)$.

$$y'(x) = \frac{z'(x)(1+x^2) - z(x)(2x)}{(1+x^2)^2}$$

$$y'(-3) = \frac{z'(-3)(1+9) - z(-3)(-6)}{(1+9)^2}$$

$$= \frac{15(10) - 6(-6)}{10^2}$$

$$= \frac{150+36}{100}$$

$$= \frac{186}{100} = \boxed{\frac{93}{50}}$$

3. $f(x) = \left(1+\sqrt{x}\right)x^3$

$$f'(x) = \left(\frac{1}{2\sqrt{x}}\right)x^3 + \left(1+\sqrt{x}\right)(3x^2)$$

$$= \frac{1}{2}x^{5/2} + 3x^2 + 3x^{5/2}$$

$$= \boxed{\frac{7}{2}x^{5/2} + 3x^2}$$

4. $g(t) = \left(\frac{2}{t} + t^5\right)(t^3+1) = (2t^{-1}+t^5)(t^3+1)$

$$g'(t) = (-2t^{-2}+5t^4)(t^3+1) + \left(\frac{2}{t}+t^5\right)(3t^2)$$

$$= \left(\frac{-2}{t^2}+5t^4\right)(t^3+1) + \left(\frac{2}{t}+t^5\right)(3t^2)$$

$$= -2t + 5t^7 - \frac{2}{t^2} + 5t^4 + 6t + 3t^7$$

$$= \boxed{8t^7 + 5t^4 + 4t - \frac{2}{t^2}}$$

5. $h(y) = \dfrac{1}{y^3+2y+1}$

$$h'(y) = \frac{0(y^3+2y+1) - 1(3y^2+2)}{(y^3+2y+1)^2}$$

$$= \boxed{\frac{-3y^2-2}{(y^3+2y+1)^2}}$$

Compute the following derivatives using

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x) \qquad \text{and} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x)$$

and the trigonometric identities

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \sec(x) = \frac{1}{\cos(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)} \qquad \csc(x) = \frac{1}{\sin(x)}$$

6. $\frac{\mathrm{d}}{\mathrm{d}x}\tan(x)$

$$\frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)}$$

$$= \boxed{\sec^2(x)}$$

7. $\frac{\mathrm{d}}{\mathrm{d}x}\cot(x)$

$$\frac{d}{dx}\frac{\cos(x)}{\sin(x)} = \frac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)}$$

$$= \frac{-(\sin^2(x)+\cos^2(x))}{\sin^2(x)}$$

$$= \frac{-1}{\sin^2(x)}$$

$$= \boxed{-\csc^2(x)}$$

8. $\frac{\mathrm{d}}{\mathrm{d}x}\sec(x)$

$$\frac{d}{dx}\frac{1}{\cos(x)} = \frac{0-(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos(x)}\frac{\sin(x)}{\cos(x)}$$

$$= \boxed{\sec(x)\tan(x)}$$

9. $\frac{\mathrm{d}}{\mathrm{d}x}\csc(x)$

$$\frac{d}{dx}\frac{1}{\sin(x)} = \frac{0-\cos(x)}{\sin^2(x)}$$

$$= -\frac{1}{\sin(x)}\frac{\cos(x)}{\sin(x)}$$

$$= \boxed{-\csc(x)\cot(x)}$$