COMPARISON TESTS

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Name: ____

Theorem (The Comparison Tests). *Let $\{a_n\}$ and $\{b_n\}$ be sequences, and assume there exists some number N such that*

$$0 < a_n \leq b_n$$

is satisfied whenever $n \geq N$.

(i) If $\sum a_n$ diverges, then $\sum b_n$ also diverges.

(ii) If $\sum b_n$ converges, then $\sum a_n$ also converges.

Theorem (The Limit Comparison Test). *Let $\{a_n\}$ and $\{b_n\}$ be sequences, and assume there exists some number N such that*

$$0 < a_n, b_n$$

is satisfied whenever $n \geq N$. If there exists some number $c > 0$ such that

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$$

then either

- *$\sum a_n$ and $\sum b_n$ both converge, or*
- *$\sum a_n$ and $\sum b_n$ both diverge.*

Decide whether the following series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{9^n}{3 + 10^n}$

2. $\displaystyle\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1+\cos(n)}{e^n}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}+2}$

5. $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$