INTEGRAL TEST AND ESTIMATES OF SUMS

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Name: ______________________________

Use the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ to answer the following questions.

1. Show that the function $f(x) = 1/x^2$ satisfies the hypotheses of the Integral Test. For n a fixed integer, compute the improper integral

$$\int_n^{\infty} \frac{1}{x^2}\,\mathrm{d}x.$$

Use this to conclude that the series $\sum_{n=1}^{\infty} 1/n^2$ converges.

2. Use a calculator to find

$$s_{10} = \sum_{n=1}^{10} \frac{1}{n^2}.$$

Use the intequality

$$\int_{11}^{\infty} \frac{1}{x^2}\,\mathrm{d}x \leq R_{10} \leq \int_{10}^{\infty} \frac{1}{x^2}\,\mathrm{d}x$$

to determine how good this estimate to the sum of the series is.

3. Use the inequality

$$s_{10} + \int_{11}^{\infty} \frac{1}{x^2}\,\mathrm{d}x \leq \sum_{n=1}^{\infty} \frac{1}{n^2} \leq s_{10} + \int_{10}^{\infty} \frac{1}{x^2}\,\mathrm{d}x$$

to find an open interval containing the number s. Compute the midpoint of this interval. Is the midpoint a better or worse approximation to the sum of the series than you found in Problem 2? Why or why not?

4. It is known that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Use this to compare your estimates from Problems 2 and 3.

5. Find the number of terms that you would need to ensure an estimate that is accurate to the first 3 decimal places.

Determine whether the following series converge or diverge.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{5n-1}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

8. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

9. $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$