## INTEGRAL TEST AND ESTIMATES OF SUMS

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Use the series  $\sum_{n=1}^{\infty} \frac{1}{n^2}$  to answer the following questions.

1. Show that the function  $f(x) = 1/x^2$  satisfies the hypotheses of the Integral Test. For n a fixed integer, compute the improper integral

$$\int_{n}^{\infty} \frac{1}{x^2} \,\mathrm{d}x.$$

Use this to conclude that the series  $\sum_{n=1}^\infty 1/n^2$  converges.

## **2.** Use a calculator to find

$$s_{10} = \sum_{n=1}^{10} \frac{1}{n^2}.$$

Use the intequality

$$\int_{11}^{\infty} \frac{1}{x^2} \, \mathrm{d}x \le R_{10} \le \int_{10}^{\infty} \frac{1}{x^2} \, \mathrm{d}x$$

to determine how good this estimate to the sum of the series is.

**3.** Use the inequality

$$s_{10} + \int_{11}^{\infty} \frac{1}{x^2} \, \mathrm{d}x \le \sum_{n=1}^{\infty} \frac{1}{n^2} \le s_{10} + \int_{10}^{\infty} \frac{1}{x^2} \, \mathrm{d}x$$

to find an open interval containing the number s. Compute the midpoint of this interval. Is the midpoint a better or worse approximation to the sum of the series than you found in Problem 2? Why or why not?

## 4. It is known that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

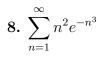
Use this to compare your estimates from Problems 2 and 3.

5. Find the number of terms that you would need to ensure an estimate that is accurate to the first 3 decimal places.

Determine whether the following series converge or diverge.

6. 
$$\sum_{n=1}^{\infty} \frac{2}{5n-1}$$

7. 
$$\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$$



9. 
$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 4}$$