INVERSE TRIG FUNCTIONS

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Name: Solutions

Compute the derivative of the given function.

1. $f(x) = \arctan(x^2)$

$$f'(x) = \frac{1}{1+(x^2)^2} \cdot 2x = \boxed{\frac{2x}{1+x^4}}$$

2. $f(x) = \arctan(x)^2$

$$f'(x) = 2\arctan(x)\left(\frac{1}{1+x^2}\right)$$

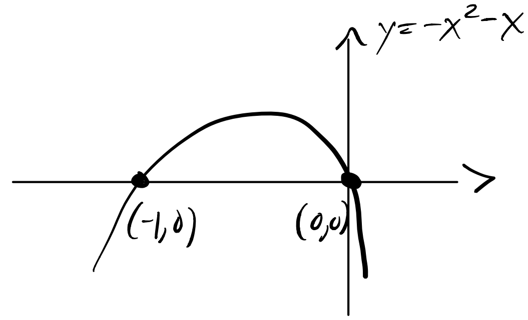
$$= \boxed{\frac{2\arctan(x)}{1+x^2}}$$

$-x^2-x = -x(x+1)$

3. $f(x) = \arcsin(2x+1)$

$$f'(x) = \frac{1}{\sqrt{1-(2x+1)^2}}\cdot 2 = \frac{2}{\sqrt{1-(4x^2+4x+1)}} = \frac{2}{\sqrt{-4x^2-4x}}$$

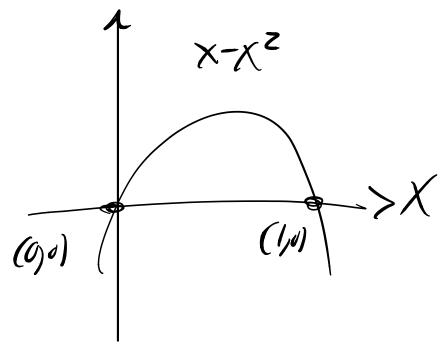
$$= \frac{2}{\sqrt{-4x(x+1)}} = \boxed{\frac{2}{\sqrt{-x(x+1)}}}, \quad -1<x<0.$$



4. $f(x) = \arccos(\sqrt{x})$

$$f'(x) = \frac{-1}{\sqrt{1-(\sqrt{x})^2}}\left(\frac{1}{2\sqrt{x}}\right)$$

$$= \boxed{\frac{-1}{2\sqrt{x(1-x)}}}, \quad 0<x<1$$



5. $f(x) = x\arcsin(x) + \sqrt{1-x^2}$

$$f'(x) = \arcsin(x) + x\left(\frac{1}{\sqrt{1-x^2}}\right) + \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$= \arcsin(x) + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$$

$$= \boxed{\arcsin(x)}.$$

Compute the following integrals

6. $\int \frac{1}{a^2+x^2}\,\mathrm{d}x$

Observe: $\frac{1}{a^2+x^2} = \frac{1}{a^2\left(1+\frac{x^2}{a^2}\right)} = \frac{1}{a^2\left(1+\left(\frac{x}{a}\right)^2\right)}$

Take $u=\frac{x}{a}$, $du=\frac{1}{a}dx$

$$\Rightarrow \int \frac{dx}{a^2+x^2} = \frac{1}{a}\int \frac{\left(\frac{1}{a}\right)dx}{1+\left(\frac{x}{a}\right)^2} = \frac{1}{a}\int \frac{du}{1+u^2} = \frac{1}{a}\arctan(u)+C$$

$$= \boxed{\frac{1}{a}\arctan\left(\frac{x}{a}\right)+C}$$

7. $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\,\mathrm{d}x$

Observe:

$$\frac{1}{\sqrt{a^2-x^2}} = \frac{1}{\sqrt{a^2\left(1-\frac{x^2}{a^2}\right)}} = \frac{1}{\sqrt{a^2}\sqrt{1-\left(\frac{x}{a}\right)^2}} = \frac{1}{a\sqrt{1-\left(\frac{x}{a}\right)^2}}$$

Take $u=\frac{x}{a}$ so $du=\frac{1}{a}dx$ and

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \int \frac{du}{\sqrt{1-u^2}}$$

$$= \arcsin(u)+C$$

$$= \boxed{\arcsin\left(\frac{x}{a}\right)+C}$$