POWER SERIES

BLAKE FARMAN

Lafayette College

Name: Solutions

For each of the power series below, find the Radius and Interval of Convergence.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2n-1}$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{2n+1} \frac{2n-1}{x^n} \right| = \lim_{n\to\infty} |x| \frac{2n-1}{2n+1} = |x| < X$$

$\underline{X = -1}$

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$ converges by the A.S.T.

$\underline{X = 1}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges by the L.C.T. with the Harmonic Series.

$R = 1,\quad [-1, 1).$

$$\frac{n^2+1}{n\sqrt{n}} = \frac{n^2+1}{n^{3/2}} \Big/ n^{1/2}$$

2. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$\lim_{n \to \infty} \left| \frac{x^{n+1}}{(n+1)!} \frac{n!}{x^n} \right| = \lim_{n \to \infty} \frac{|x|}{n+1} = 0$$

$R = \infty$, $(-\infty, \infty)$

3. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(n+1)^2+1} \frac{n^2+1}{(x-2)^n} \right| = \lim_{n\to\infty} |x-2| \frac{n^2+1}{(n+1)^2+1} = |x-2| < 1$$

<u>$x=3$</u>

$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges by L.C.T. with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

<u>$x=1$</u>

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$ converges absolutely (above)

$R=1$, $[1,3]$.

4. $\displaystyle\sum_{n=\cancel{1}2}^{\infty} \frac{(x+2)^n}{2^n \ln(n)}$

$$\lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{2^{n+1}\ln(n+1)} \frac{2^n \ln(n)}{(x+2)^n} \right| = \lim_{n\to\infty} \frac{|x+2|}{2} \frac{\ln(n)}{\ln(n+1)} = \frac{|x+2|}{2} < 1$$

$$\Rightarrow |x+2| < 2$$

$\underline{x=-4}$

$$\sum_{n=2}^{\infty} \frac{(-2)^n}{2^n \ln(n)} = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$ converges by the A.S.T.

$\underline{x=0}$

$$\sum_{n=2}^{\infty} \frac{2^n}{2^n \ln(n)} = \sum_{n=2}^{\infty} \frac{1}{\ln(n)}$$ diverges by Comparison with the Harmonic Series.

<u>Observe</u>: $2 < e \Rightarrow \ln(2) < \ln(e) = 1 < 2$

$\Rightarrow \ln(2) - 2 < 0$

and

$$\frac{d}{dx}\left[\ln(x) - x\right] = \frac{1}{x} - 1 < 0 \quad \text{for} \quad x \geq 2$$

yields the inequality

$$\ln(n) - n < 0 \Rightarrow \ln(n) < n \Rightarrow \frac{1}{n} < \frac{1}{\ln(n)} \quad \text{for all } n \geq 2$$

$R = 2$, $[-4, 0)$.