RATIO AND ROOT TESTS

BLAKE FARMAN

Lafayette College

Name: Solutions

Determine whether the following series converge or diverge.

1. $\sum_{n=1}^{\infty} \frac{n}{5^n}$ Converges by the Ratio Test:

$$\lim_{n\to\infty} \frac{n+1}{5^{n+1}} \frac{5^n}{n} = \lim_{n\to\infty} \frac{n+1}{5n} = \frac{1}{5} < 1$$

2. $\sum_{k=1}^{\infty} \frac{1}{k!}$ Converges by the Ratio Test:

$$\lim_{k\to\infty} \frac{k!}{(k+1)!} = \lim_{k\to\infty} \frac{1}{k+1} = 0 < 1$$

3. $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$ Converges by the Ratio Test

$$\lim_{n\to\infty} \frac{10^{n+1}}{(n+2)4^{2n+3}} \frac{(n+1)4^{2n+1}}{10^n} = \lim_{n\to\infty} \frac{10}{4^2}\frac{n+1}{n+2} = \frac{10}{16} < 1$$

4. $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$ Converges by the Root Test

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n^2+1}{2n^2+1}\right)^n} = \lim_{n\to\infty} \frac{n^2+1}{2n^2+1} = \frac{1}{2} < 1$$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(\ln(n))^n}$ Converges by the Root Test:

$$\lim_{n\to\infty} \sqrt[n]{\frac{1}{\ln(n)^n}} = \lim_{n\to\infty} \frac{1}{\ln(n)} = 0 < 1.$$

6. $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$ Diverges by the Root Test

$$\lim_{n\to\infty} \sqrt[n]{\left(1+\frac{1}{n}\right)^{n^2}} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e > 1.$$