SEQUENCES

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Name: Solutions

List the first five terms of the sequence.

1. $a_n = \dfrac{2^n}{2n+1}$

$\frac{2}{3}, \frac{4}{5}, \frac{8}{7}, \frac{16}{9}, \frac{32}{11}$

2. $a_n = \dfrac{n^2-1}{n^2+1}$

$0, \frac{3}{5}, \frac{8}{10}, \frac{15}{17}, \frac{24}{26}$

3. $a_n = \dfrac{1}{(n+1)!}$

$\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \frac{1}{720}$

4. $a_1 = 1$, $a_{n+1} = 5a_n - 3$

$1, 2, 7, 32, 157$

Find a formula for the general term of the sequence $\{a_n\}_{n=1}^{\infty}$, assuming that the pattern continues.

5. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \dots\right\}$

$$a_n = \boxed{\frac{1}{2n}}$$

6. $\left\{4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \dots\right\}$

$$a_n = (-1)^{n-1}\left(\frac{1}{4}\right)^{n-2} = \frac{(-1)^{n-1}}{4^{n-2}} = \boxed{4\left(\frac{-1}{4}\right)^{n-1}}$$

7. $\left\{-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \dots\right\}$

$$a_n = (-1)^n\left(\frac{1}{3}\right)^{n-2} 2^{n-1} = \frac{(-1)^n 2^{n-1}}{3^{n-2}}\left(\frac{3}{3}\right) = \frac{-3(-1)^{n-1} 2^{n-1}}{3^{n-1}}$$

$$= \boxed{-3\left(\frac{-2}{3}\right)^{n-1}}$$

8. $\{5, 8, 11, 14, 17, \dots\}$

$$a_n = 5 + 3(n-1) = 5 + 3n - 3 = \boxed{3n+2}$$

9. $\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \dots\right\}$

$$\boxed{a_n = (-1)^{n+1}\frac{n^2}{n+1}}$$

10. $\{1, 0, -1, 0, 1, 0, -1, 0, \dots\}$

$$\boxed{a_n = \sin\left(\frac{n\pi}{2}\right) = \cos\left((n-1)\frac{\pi}{2}\right)}$$

Determine whether the sequence converges or diverges. If it converges, find the limit.

11. $a_n = \dfrac{3+5n^2}{n+n^2}$

$$\lim_{n\to\infty} \frac{3+5n^2}{n+n^2} \overset{L'H}{=} \lim_{n\to\infty} \frac{10n}{2n+1}$$

$$\overset{L'H}{=} \lim_{n\to\infty} \frac{10}{2}$$

$$= \boxed{5}$$

12. $a_n = \dfrac{n^4}{n^3-2n}$

$$\lim_{n\to\infty} \frac{n^4}{n^3-2n} \overset{L'H}{=} \lim_{n\to\infty} \frac{4n^3}{3n^2-2}$$

$$\overset{L'H}{=} \lim_{n\to\infty} \frac{12n^2}{6n}$$

$$= \lim_{n\to\infty} 2n$$

$$= \boxed{\infty}$$

13. $a_n = 3^n 7^{-n}$

$$\lim_{n\to\infty} \frac{3^n}{7^n} = \lim_{n\to\infty} \left(\frac{3}{7}\right)^n$$

$$= \lim_{n\to\infty} e^{\ln(3/7)n}$$

$$= \boxed{0}$$

Since $\ln(3/7) < \ln(1) = 0$.

14. $a_n = \sqrt{\dfrac{1+4n^2}{1+n^2}}$

$$\lim_{n\to\infty} \frac{1+4n^2}{1+n^2} \overset{L'H}{=} \lim_{n\to\infty} \frac{8n}{2n} = \lim_{n\to\infty} 4 = 4$$

so

$$\lim_{n\to\infty} \sqrt{\frac{1+4n^2}{1+n^2}} = \sqrt{\lim_{n\to\infty} \frac{1+4n^2}{1+n^2}} = \sqrt{4} = \boxed{2}$$

15. $a_n = \dfrac{3\sqrt{n}}{\sqrt{n}+2}$

$$\lim_{n\to\infty} \frac{3\sqrt{n}}{\sqrt{n}+2} \overset{L'H}{=} \lim_{n\to\infty} \frac{3/2\sqrt{n}}{1/2\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{3}{2\sqrt{n}} 2\sqrt{n}$$

$$= \lim_{n\to\infty} 3$$

$$= \boxed{3}$$

16. $a_n = \cos\left(\dfrac{n\pi}{n+1}\right)$

$$\lim_{n\to\infty} \frac{n\pi}{n+1} \overset{L'H}{=} \lim_{n\to\infty} \frac{\pi}{1} = \pi$$

$$\Rightarrow \lim_{n\to\infty} \cos\left(\frac{n\pi}{n+1}\right) = \cos\left(\lim_{n\to\infty} \frac{n\pi}{n+1}\right) = \cos(\pi) = \boxed{-1}$$