TAYLOR SERIES

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Name: Solutions

Find the Taylor series expansion for the function centered at a.

1. $f(x) = x \ln(1 + 2x)$, $a = 0$

$$f(x) = x \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2x)^n}{n}$$

$$= \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^n}{n} x^{n+1} \qquad |x| < \frac{1}{2}}$$

2. $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$

$$= \ln(1+x) - \ln(1-x)$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n - \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (-x)^n$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} - (-1)^{n-1}(-1)^n}{n} x^n$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} + (-1)^{2n}}{n} x^n$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} + 1}{n} x^n$$

$$= \boxed{\sum_{n=1}^{\infty} \frac{2}{2n-1} x^{2n-1}, \quad |x| < 1}$$

Since

$$\frac{(-1)^{n-1} + 1}{n} = \begin{cases} \frac{1+1}{n} = \frac{2}{n} & \text{if } n \text{ odd} \\ \frac{-1+1}{n} = 0 & \text{if } n \text{ even} \end{cases}$$

3. $f(x) = \dfrac{1}{x}$, $a = 1$

$$\frac{1}{x} = \frac{1}{1-(1-x)} = \sum_{n=0}^{\infty} (1-x)^n$$

$$= \sum_{n=0}^{\infty} (-(x-1))^n$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n (x-1)^n, \quad |x-1| < 1}$$

4. $f(x) = \ln(x)$, $a = 1$

$$\ln(x) + C = \int \frac{1}{x}\,dx$$

$$= \int \sum_{n=0}^{\infty} (-1)^n (x-1)^n \,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \int (x-1)^n \,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{n+1}$$

$$= \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x-1)^n}{n}, \quad |x-1| < 1}$$