EXAM 1
MATH 161

BLAKE FARMAN

Lafayette College

Answer the questions in the spaces provided on the question sheets and turn them in at the end of the exam period.
It is advised, although not required, that you check your answers. All supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**
You may **not** use a calculator or any other electronic device, including cell phones, smart watches, etc.
By writing your name on the line below, you indicate that you have read and understand these directions.

Name: Solutions

Problem	Points Earned	Points Possible
1		6
2		2
3		1
4		8
5		5
6		5
7		3
8		10
9		10
10		5
11		15
12		15
13		15
Total		100

Date: September 26, 2018.

Fill In the Blank

1 (6 Points - Limit Laws)**.** Suppose that c is a constant and the limits

$$\lim_{x\to a} f(x) = L \text{ and } \lim_{x\to a} g(x) = M$$

exist. Then

(1) $\lim_{x\to a}\big[f(x)+g(x)\big] =$ $L+M$

(2) $\lim_{x\to a}\big[f(x)-g(x)\big] =$ $L-M$

(3) $\lim_{x\to a}[c(fx)] =$ cL

(4) $\lim_{x\to a}\big[f(x)g(x)\big] =$ LM

(5) $\lim_{x\to a}\frac{f(x)}{g(x)} =$ L/M, if $M\neq 0$

2 (2 Points)**.** A function f is continuous at a number a if

$$\lim_{x\to a} f(x) = f(a).$$

3 (1 Point)**.** The derivative of the function $f(x)$ is the function

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

4 (8 Points - Derivative Rules)**.** Let c be a constant. If f and g are differentiable functions, then

(1) $\frac{\mathrm{d}}{\mathrm{d}x}(c) =$ 0

(2) $\frac{\mathrm{d}}{\mathrm{d}x}(x^n) =$ nx^{n-1}

(3) $\frac{\mathrm{d}}{\mathrm{d}x}\big(cf(x)\big) =$ $cf'(x)$

(4) $\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)+g(x)\big) =$ $f'(x)+g'(x)$

(5) $\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)-g(x)\big) =$ $f'(x)-g'(x)$

(6) $\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)g(x)\right) =$ $f'(x)g(x)+f(x)g'(x)$

(7) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) =$ $\frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$

(8) $\frac{\mathrm{d}}{\mathrm{d}x}\left(f\circ g(x)\right) = \frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) =$ $f'(g(x))g'(x)$

Problems

Use the function $f(x) = \dfrac{x^2-9}{x^2+2x-3}$ to answer Problems 5-7.

5 (5 Points)**.** Use the Limit Laws to compute $\lim_{x\to -3} f(x)$.

$$\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3} = \lim_{x\to -3}\frac{(x+3)(x-3)}{(x-1)(x+3)} = \lim_{x\to -3}\frac{x-3}{x-1} = \frac{-3-3}{-3-1} = \frac{-6}{-4} = \boxed{\frac{3}{2}}$$

6 (5 Points)**.** Use the Limit Laws to compute $\lim_{x\to 1} f(x)$.

Observe that when $0<x<1$, $-3<x-3<-2$ and $-1<x-1<0$, so $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\frac{x-3}{x-1} = \infty$. When $1<x<2$, $-2<x<-1$ and $0<x-1<1$, so $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\frac{x-3}{x-1} = -\infty$. Therefore $\lim_{x\to 1} f(x)$ does not exist.

7 (3 Points)**.** Find the value L that makes the function

$$g(x) = \begin{cases} f(x) & \text{if } x \neq -3, \\ L & \text{if } x = -3 \end{cases}$$

continuous at $x = -3$.

To be continuous, we just need

$$L = g(-3) = \lim_{x\to -3} g(x) = \lim_{x\to -3} f(x) = \boxed{3/2}$$

8 (10 Points)**.** Use the Limit Laws to compute

$$\lim_{x\to 16} \frac{4-\sqrt{x}}{x-16}.$$

$$\lim_{x\to 16} \frac{4-\sqrt{x}}{x-16} = \lim_{x\to 16} \frac{(4-\sqrt{x})(4+\sqrt{x})}{(x-16)(4+\sqrt{x})}$$

$$= \lim_{x\to 16} \frac{16-x}{(x-16)(4+\sqrt{x})}$$

$$= \lim_{x\to 16} \frac{-1}{4+\sqrt{x}}$$

$$= \frac{-1}{4+\sqrt{16}} = \frac{-1}{4+4} = \boxed{\frac{-1}{8}}$$

Use $f(x) = 3x^2 + 1$ to answer Problems 9 and 10.

9 (10 Points)**.** Compute $f'(x)$ by the **definition**.

$$f'(x) = \lim_{h\to 0} \frac{3(x+h)^2+1-(3x^2+1)}{h} = \lim_{h\to 0} \frac{3(x^2+2xh+h^2)+1-3x^2-1}{h}$$

$$= \lim_{h\to 0} \frac{3x^2-3x^2+6xh+3h^2+1-1}{h}$$

$$= \lim_{h\to 0} \frac{6xh+3h^2}{h}$$

$$= \lim_{h\to 0} 6x+3h = \boxed{6x.}$$

10 (5 Points)**.** Use the derivative rules to check that your answer to part (a) is correct.

$$\frac{d}{dx}(3x^2+1) = \frac{d}{dx}(3x^2) + \frac{d}{dx}(1) = 3\frac{d}{dx}(x^2) + \frac{d}{dx}(1) = 3(2x)+0 = \boxed{6x}$$

In the following problems, find the equation of the line tangent to the given curve at the given point. **Do not compute the derivative from the definition!**

11 (15 Points). $f(x) = (x^4 - 3x^2 + 5)^3$; $(0, 125)$.

$f'(x) = 3(x^4 - 3x^2 + 5)^2(4x^3 - 6x)$

$f'(0) = 3(0 - 0 + 5)^2(0 - 0) = 0$

$y - 125 = 0(x - 0) = 0$

$\Rightarrow \boxed{y = 125}$

This is clearly wrong!

12 (15 Points). $g(x) = \dfrac{x}{1 - x^2}$; $(2, 1)$. $g(2) = \frac{2}{1-2^2} = \frac{2}{1-4} = \frac{2}{-3} = -\frac{2}{3}$, not 1.

$g'(x) = \dfrac{(1)(1 - x^2) - x(-2x)}{(1 - x^2)^2} = \dfrac{1 - x^2 + 2x^2}{(1 - x^2)^2} = \dfrac{1 + x^2}{(1 - x^2)^2}$

$g'(2) = \dfrac{1 + 2^2}{(1 - 2^2)^2} = \dfrac{1 + 4}{(1 - 4)^2} = \dfrac{5}{3^2} = \dfrac{5}{9}$

$y - \frac{2}{3} = 5/9(x - 2) \Rightarrow y = 5/9x - 10/9 + \frac{6}{9} = \boxed{5/9x - 4/9}$

13 (15 Points). $h(x) = x^3 \sin(2x)$; $(\pi, 0)$.

$h'(x) = 3x^2 \sin(2x) + x^3 \cos(2x)(2) = 3x^2 \sin(2x) + 2x^3 \cos(2x)$

$h'(\pi) = 3\pi^2 \sin(2\pi) + 2\pi^3 \cos(2\pi) = 0 + 2\pi^3 = 2\pi^3.$

$y - 0 = 2\pi^3(x - \pi) = \boxed{2\pi x^3 - 2\pi^4}$