EXAM 1

BLAKE FARMAN

Lafayette College

Answer the questions in the spaces provided on the question sheets and turn them in at the end of the exam period.
It is advised, although not required, that you check your answers. All supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**
You may **not** use a calculator or any other electronic device, including cell phones, smart watches, etc.

Name: Solutions

Date: February 20, 2019.

Functions

1. Compute

$$\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4x+4}$$

$$\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4x+4} = \lim_{x\to 2} \frac{(x-2)(x-3)}{(x-2)^2} = \lim_{x\to 2} \frac{x-3}{x-2}$$

$\lim_{x\to 2^+} \frac{x-3}{x-2} = -\infty$ because $x-3<0$ and $0<x-2$ as x approaches 2 from the right.

$\lim_{x\to 2^-} \frac{x-3}{x-2} = \infty$ because $x-3<0$ and $x-2<0$ as x approaches 2 from the left.

Therefore $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4x+4}$ $\boxed{\text{Does Not Exist!}}$

2. Find the value of b that makes the function

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } 2 < x \\ 2x+b & \text{if } x \leq 2 \end{cases}$$

a continuous function.

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x^2-4}{x-2} = \lim_{x\to 2^+} \frac{(x-2)(x+2)}{(x-2)} = \lim_{x\to 2^+} x+2 = 2+2 = 4.$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2x+b = 2(2)+b = 4+b$$

To be continuous we must have

$$f(2) = 4+b = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 4$$

Therefore $\boxed{b=0}$

Derivatives

3. Use the **limit definition**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to compute the derivative of

$$f(x) = \sqrt{x}.$$

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$= \boxed{\frac{1}{2\sqrt{x}}}$$

Derivative Rules

4. Compute the line tangent to

$$f(x) = 2x^3 - 3x^2 - 4x$$

at the point $(-1, -1)$.

$$f'(x) = 2(3x^2) - 3(2x) - 4(1)$$
$$= 6x^2 - 6x - 4$$

$$f'(-1) = 6(-1)^2 - 6(-1) - 4$$
$$= 6 + 6 - 4$$
$$= 8.$$

$$y - (-1) = 8(x - (-1))$$
$$\Rightarrow \boxed{y + 1 = 8(x+1)} \quad \text{or} \quad \boxed{y = 8x + 7}$$

Product and Quotient Rules

5. Compute
$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\cos(x)\sin(x)\right].$$

$$\frac{d}{dx}\left[\cos(x)\sin(x)\right] = \frac{d}{dx}\left[\cos(x)\right]\sin(x) + \cos(x)\frac{d}{dx}\left[\sin(x)\right]$$
$$= (-\sin(x))\sin(x) + \cos(x)(\cos(x))$$
$$= \boxed{-\sin^2(x) + \cos^2(x).}$$

6. Compute
$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x^2+3}{\cos(x)}\right].$$

$$\frac{d}{dx}\left[\frac{x^2+3}{\cos(x)}\right] = \frac{\frac{d}{dx}\left[x^2+3\right]\cos(x) - (x^2+3)\frac{d}{dx}\left[\cos(x)\right]}{\cos^2(x)}$$
$$= \frac{2x\cos(x) - (x^2+3)(-\sin(x))}{\cos^2(x)}$$
$$= \boxed{\frac{2x\cos(x) + (x^2+3)\sin(x)}{\cos^2(x)}}$$

Chain Rule

7. Compute

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan\left(3x^2+4x+5\right)\right].$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}(3x^2+4x+5) = 6x+4$$

$$\frac{d}{dx}\tan(3x^2+4x+5) = \boxed{\sec^2(3x^2+4x+5)(6x+4).}$$