AREA AND VOLUME

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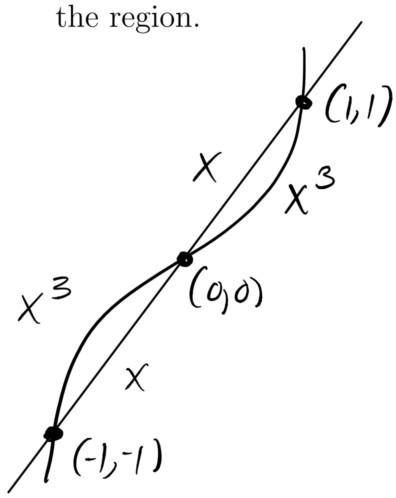
Name: Solutions

1. Sketch the region enclosed by the curves $y = x^3$ and $y = x$, and compute the area of the region.



$$\int_{-1}^{0} [x^3 - x]\,dx + \int_{0}^{1} [x - x^3]\,dx = \frac{1}{4}x^4\Big|_{-1}^{0} - \frac{1}{2}x^2\Big|_{-1}^{0} + \frac{1}{2}x^2\Big|_{0}^{1} - \frac{1}{4}x^4\Big|_{0}^{1}$$

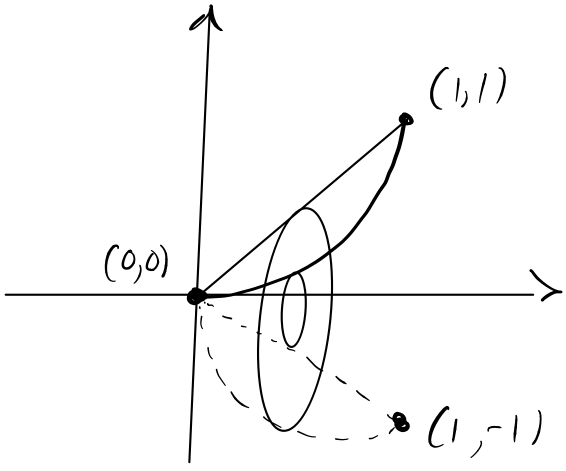
$$= \frac{1}{4}(0-1) - \frac{1}{2}(0-1) + \frac{1}{2}(1-0) - \frac{1}{4}(1-0)$$

$$= -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$$

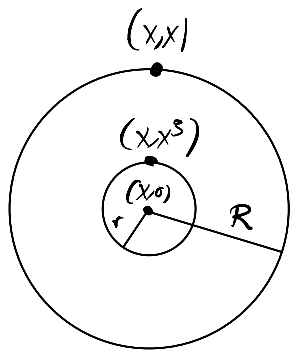
$$= 1 - \frac{1}{2}$$

$$= \boxed{\frac{1}{2}}$$

2. Find the volume of the solid obtained by rotating the region bounded by the given curves $y = x^3$, $y = x$, $0 \le x$ about the x-axis. Sketch a typical cross section of the solid.



Cross-Section



$R = x - 0 = x$

$r = x^3 - 0 = x^3$

$$A(x) = \pi R^2 - \pi r^2 = \pi\left(x^2 - x^6\right)$$

$$\text{Volume} = \int_0^1 A(x)\,dx = \pi\int_0^1 (x^2 - x^6)\,dx$$

$$= \pi\left(\frac{1}{3}x^3\Big|_0^1 - \frac{1}{7}x^7\Big|_0^1\right)$$

$$= \pi\left(\frac{1}{3}(1-0) - \frac{1}{7}(1-0)\right)$$

$$= \pi\left(\frac{7-3}{21}\right)$$

$$= \boxed{\frac{4\pi}{21}}$$