RATIO AND ROOT TEST QUIZ

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Name: Solutions

Determine whether the following series converge or diverge.

1. $\sum_{n=1}^{\infty} \frac{(-9)^n}{n10^{n+1}}$ Converges Absolutely by the Ratio Test:

$$\lim_{n\to\infty} \left| \frac{(-9)^{n+1}}{(n+1)10^{n+2}} \cdot \frac{n10^{n+1}}{(-9)^n} \right| = \lim_{n\to\infty} \frac{9n}{10(n+1)}$$

$$= \frac{9}{10} < 1$$

2. $\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$ Diverges by the Root Test:

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{-2n}{n+1}\right|^{5n}} = \lim_{n\to\infty} \left|\frac{-2n}{n+1}\right|^{5}$$

$$= \lim_{n\to\infty} \left(\frac{2n}{n+1}\right)^{5}$$

$$= \left(\lim_{n\to\infty} \frac{2n}{n+1}\right)^{5}$$

$$= 2^5 = 32 > 1.$$